STUDIES OF ARTERIAL INPUT IMPEDANCE IN MODELS OF CAROTID ARTERY

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Abstract. A non-invasive method of measurement and analysis of the arterial input impedance of the carotid artery was applied to silicone models of the normal and stenosed carotid bifurcation. The experimental setup enabled the simulation of the “in vivo” conditions. The experimental results were compared with impedance computed for a lumped element electrical model and a model containing transmission line elements. The stenosis resulted in increased impedance moduli and phase values. Similar phenomena were observed for both computational results; however, the transmission line model yielded phase plots closer to the experimental one. The method of measurement and analysis of the arterial input impedance appears to be an efficient tool for the assessment of the properties of the carotid bifurcation.

1 INTRODUCTION

The arterial input impedance is a source of information on the properties of the arterial bed distal to the measurement point. The arterial input impedance of the carotid artery may give information on the presence and severity of a stenosis, important when the narrowing is not accessible using Doppler or echographic techniques. This impedance may be important for the assessment of the effect of drugs and for the follow-up after a surgery.

This study is a part of the validation procedure of an indirect method of the measurement of the input impedance of the carotid artery applied in clinic[1, 2]. The impedance is computed from volume flow data obtained from the ultrasonic Doppler velocity signal and the pressure signal, obtained non-invasively by ultrasonic tracking of the carotid wall calibrated with the Riva-Rocci-Korotkoff pressure measurement[2, 3]. It is of interest to validate this approach in controlled experimental conditions, where the geometry of the model is known, the hydraulic resistance of the narrowing may be measured, as well as the pressure within the model, and to compare the experimental results with results obtained from lumped electrical equivalent circuit and circuit comprising transmission line elements.

2 METHODS

2.1 Experimental setup

1:1 scale silicone rubber (Silgel 601, Wacker, FRG) replicas of carotid bifurcations were made using the lost wax technique[4]. Built were normal model and model with internal carotid artery (ICA) stenosis denoted as 90%, subsequently assessed with
impedance measurement for constant components of flow and pressure (Fig.1).

Fig.1. Drawing of a model of the normal carotid bifurcation (left) and bifurcation with 90% stenosis in the ICA branch (right). The dimensions of the parts indicated are specified in the Table1.

The flow simulation setup provided pulsatile flow, generated by the computer controlled piston pump SPS3891 (Vivitro Systems, Canada) (Fig.2), and allowed to obtain the typical flow conditions seen in carotid arteries\(^5\), e.g. Reynolds numbers in the CCA branch in the range 250 - 400 and the Womersley number about 4 (1Hz cycle). A 1:1 by weight mixture of water and glycerine, with a density of about 1100kg/m\(^3\) and viscosity of 4-5mPas, was used as the experimental liquid. The liquid was seeded with BASF disc-shaped particles, having a diameter of about 7\(\mu\)m\(^6\).

Fig.2. Schematic diagram of the experimental setup. 1 – movable overflow tank, 2 - computer controlled (9) pump (SPS3891, Vivitro Systems, Canada), 3 – artificial cardiac valves, 4 compliances (aortic, ECA and ICA load), 5 – container with the bifurcation model, 6 – hydraulic resistors, 7 – collecting tank, 8 – pump, 10 - instrumentation (see text for details).

An arterial input impedance includes the compliant, resistive and inertial elements\(^7-10\) and is usually represented using a 4-element model (Fig.3). In the stand the compliant and resistive elements were simulated using an air container and a laminar hydraulic resistor, whereas the inertance resulted from the presence of liquid in the tubings.

The resistors were built by pouring a polymerizing agent (PX 515, Axson,
France) into a container with nylon line pieces stretched inside\cite{11}, having a diameter 0.15cm. The number of bores in each resistor was 8, the length of the resistors varied from 17cm to 19cm. The ICA resistor used 4 bores and the ECA resistor used 2 bores. The computed resistance values equalled $17 \cdot 10^8$ (ICA branch) and $38 \cdot 10^8 [kg \ m^{-4} s^{-1}]$ (ECA branch), and were close to the literature data concerning the ICA and ECA loads\cite{12, 13}. These values were very close to those obtained in the experimental setup for constant components of flow and pressure, e.g. the measurements of the ICA resistor for Reynolds number in the range 250-420, viscosity 5mPas, resulted in a value of $17.6 \pm 0.27 \cdot 10^8 [kg \ m^{-4} s^{-1}]$.

The 3cm diameter, 18cm height cylinders were used as load compliances \cite{11}. The compliance values were equal to the ratio of the air volume above the liquid level to the pressure in this volume. The volumes corresponding to compliance values observed in the human carotid arteries amount to 1 - 2cm height of the air inside the cylinders (7-14cm$^3$).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{4-element circuit modelling the arterial input impedance of the CCA. Rp – peripheral resistance, Ro – serial resistance, L – inertance, C – compliance.}
\end{figure}

### 2.2 Data collection and analysis

The ultrasonic Doppler velocity and wall displacement data were collected in the CCA part of the model, 5cm proximal to the bifurcation, using the specialized instrument VED comprising an ultrasonic PW wall tracking system working at 6.75MHz and a CW Doppler apparatus working at 4.5MHz, providing flow velocity signal\cite{2}. The instrument was equipped with a double probe allowing to measure the displacement of the model wall and the flow velocity in the same area. The Doppler signals were submitted to a real–time spectral analysis by a dedicated hardware.

The internal pressure data were collected using a tip-catheter manometer (ANP-529A, Sentron, Nl), operating with a laboratory built instrumentation amplifier. The pressure acquisitions were carried out in the CCA branch 5cm distal to the bifurcation and between the load resistor and air container in each branch.

The wall tracking information, Doppler spectra and the pressure signal from the amplifier were transferred to a PC computer. The dedicated software allowed the computation of the volume flow curve using the Doppler spectral data and diameter information from the wall tracking system. Subsequently the pressure and volume flow curves were averaged (10-12 averagings) and submitted to the Fourier spectral analysis. The arterial input impedance was computed from spectral components of the pressure and volume flow curve.

### 2.3 Impedance modelling

The impedance of the part of the experimental stand distal to the measurement point was represented by an electrical equivalent circuit (Fig.4a), containing two parts corresponding to the ICA and ECA branches of the model together with the load of each branch. The inertance values were computed from the geometry of the model.
and the stand (Fig.4b, Tables 1 and 2). The values of the serial viscous resistances and load resistances were obtained from measurements for the constant components of flow and pressure (Table 2). The adequacy of this circuit as an equivalent of the impedance of the bifurcation model was assessed comparing the response of this circuit excited with the experimental pressure with the experimentally recorded volume flow curve. The Pearson $R^2$ coefficient was used as a measure of goodness of fit.

**Table 1.**
Geometry of the models and connecting elements of the stand and computed values of the inertances. Density 1120kg/m$^3$.

<table>
<thead>
<tr>
<th>Part</th>
<th>Length [mm]</th>
<th>Diameter [mm]</th>
<th>Inertance $10^5$[kg m$^{-4}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCA</td>
<td>70</td>
<td>7.15</td>
<td>19.5957</td>
</tr>
<tr>
<td>ICA</td>
<td>35</td>
<td>5.30</td>
<td>17.8316</td>
</tr>
<tr>
<td>ECA</td>
<td>25</td>
<td>4.50</td>
<td>17.6681</td>
</tr>
<tr>
<td>CCA 1</td>
<td>22</td>
<td>5.30</td>
<td>11.2084</td>
</tr>
<tr>
<td>ICA 1</td>
<td>6</td>
<td>1.94</td>
<td>22.8151</td>
</tr>
<tr>
<td>ICA 2</td>
<td>10</td>
<td>2.30</td>
<td>27.0533</td>
</tr>
<tr>
<td>ICA 3</td>
<td>22</td>
<td>3.70</td>
<td>22.9982</td>
</tr>
<tr>
<td>ECA 1</td>
<td>9</td>
<td>4.00</td>
<td>8.0500</td>
</tr>
<tr>
<td>ECA 2</td>
<td>19</td>
<td>3.60</td>
<td>20.9809</td>
</tr>
<tr>
<td>plexi tube1</td>
<td>40</td>
<td>4.00</td>
<td>35.7780</td>
</tr>
<tr>
<td>plexi tube2</td>
<td>200</td>
<td>7.00</td>
<td>58.4131</td>
</tr>
<tr>
<td>PVC tube 1</td>
<td>215</td>
<td>9.80</td>
<td>32.0378</td>
</tr>
<tr>
<td>plexi tube 3</td>
<td>100</td>
<td>8.00</td>
<td>22.3612</td>
</tr>
<tr>
<td>PVC tube 2</td>
<td>40</td>
<td>7.00</td>
<td>11.6826</td>
</tr>
</tbody>
</table>

**Table 2.**
Inertances, hydraulic serial resistance $R_s$ and load resistance $R_l$ present in the ICA and ECA branches of the models. Viscosity 5.33mPas.

<table>
<thead>
<tr>
<th>Model</th>
<th>Total inertance $10^5$[kg m$^{-4}$]</th>
<th>$R_s$ $10^8$[kg m$^{-4}$ s$^{-1}$]</th>
<th>$R_l$ $10^8$[kg m$^{-4}$ s$^{-1}$]</th>
<th>Total resistance for constant flow, $10^8$[kg m$^{-4}$ s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal model</td>
<td>ICA 197.7003</td>
<td>0.5702</td>
<td>19.7304</td>
<td>13.73</td>
</tr>
<tr>
<td></td>
<td>ECA 199.0269</td>
<td>0.6958</td>
<td>41.7225</td>
<td></td>
</tr>
<tr>
<td>Stenosed model</td>
<td>ICA 264.4317</td>
<td>4.4074</td>
<td>19.8100</td>
<td>15.49</td>
</tr>
<tr>
<td></td>
<td>ECA 222.0861</td>
<td>0.4073</td>
<td>42.6110</td>
<td></td>
</tr>
</tbody>
</table>
The electrical equivalent circuit of the hydraulic stand, comprising RLC elements present in the ICA and ECA branches, computed on the basis of the stand geometry (inertances, compliances) and of the measurements for constant components of pressure and flow (resistances). $R \ [\text{kg m}^{-4}\text{s}^{-1}]$, $C \ [\text{kg}^{-1}\text{m}^4\text{s}^2]$, $L \ [\text{kg m}^{-4}]$. The values in the figure apply to the normal (non-stenosed) model, liquid viscosity of 5.33 mPas, 1.5 cm air height in the containers. The diagram accounts for the inertance of the hydraulic resistors and compliance of tubing’s allowing to control the amount of air in the containers. b) Diagram of connecting elements between the bifurcation model and the load.

For the purpose of comparison the input impedance of another electrical equivalent circuit of the measurement stand, comprising transmission line elements, was also computed. Specifically, only the PVC tube 1 and the model itself were considered as transmission line elements. The other PVC element was treated as a lumped one because of its relatively large diameter and short length (Table 1). Therefore, the lumped load consisting of the resistor, air container, short PVC tube 2 and plexi tube 3, was connected to the first transmission line element – the PVC tube 1. It was assumed that the first reflection phenomenon occurred at this connection point. The following plexi tubes 1 and 2 were treated as serial resistors. The following connection with the bifurcation branch was considered the next reflection site. In the case of the stenosed model the branches were divided into smaller parts according to their diameters (Fig.1, Table 1). The input impedances of the ICA and ECA branches were added in parallel. The bifurcation itself was considered the next reflection site. Subsequently the impedance in the CCA branch, 5 cm proximal to the bifurcation, was computed. The computations were based on the Womersley theory, and the following formulae were used:\(^8\) :

\[
Z_c = \frac{\rho c}{\pi r^2} \sqrt{\frac{1}{1 - \sigma^2}} M_{10}^{1/2} \exp(-j \epsilon_{10} / 2)
\]

(1)
\[
\gamma = \frac{j\omega \sqrt{1 - \sigma^2}}{cM^{\frac{1}{2}}_{10}} \exp(-j\varepsilon_{10}/2) 
\]
(2)

\[
\Gamma = \frac{Z_L - Z_{sc}}{Z_L + Z_{sc}} 
\]
(3)

\[
Z_{sc} = Z_c \frac{1 + \Gamma \exp(-2\gamma l)}{1 - \Gamma \exp(-2\gamma l)} 
\]
(4)

\[
c = \sqrt \frac{Eh}{2\rho} 
\]
(5)

where \(Z_{sc}\), \(Z_{we}\) i \(Z_L\) denote respectively the characteristic impedance, the arterial input impedance of a segment, the load impedance of a segment, \(\omega\), \(\rho\), \(\sigma\), \(c\), \(r\), \(h\) – respectively the angular frequency, the liquid density, the Poisson ratio (0.48 for the silicone and 0.45 for the PVC were adopted), the wave propagation velocity computed from the Moens-Korteweg equation (5), the radius of the segment analysed, the wall thickness of a given segment (1.5mm PVC tube 2, 0.5mm bifurcation model), \(\gamma\) – the propagation coefficient, \(\Gamma\) – the reflection coefficient, \(M_{10}\) i \(\varepsilon_{10}\) – the tabulated values of the Womersley functions of the \(\alpha\) parameter\[8\].

Subsequently a 4-elements lumped electrical model, comprising the serial and peripheral resistance, capacitance and inductance (Fig.3), was fitted to the experimental impedance data. The following set of equations was used in these computations\[1\]:

\[
R_s + R_p = \text{abs}(Z_0) 
\]
(6)

\[
R_s + \frac{R_p}{1 + 2\pi f_o CR_p} = \text{Re}(Z_t) 
\]
(7)

\[
2\pi f_o L - \frac{2\pi f_o CR_p^2}{1 + (2\pi f_o CR_p^2)^2} = \text{Im}(Z_t) 
\]
(8)

\[
L = \frac{CR_p^2}{1 + (2\pi f_m CR_p^2)^2} 
\]
(9)

where \(Z_0\) and \(Z_t\) denote the zeroth and the first component of the input impedance from the experiment, \(R_s\) and \(R_p\) – serial and peripheral resistance, \(C\), \(L\) – compliance and inertance, \(f_o\) and \(f_m\) – the fundamental cycle frequency and the frequency of the intersection of the phase plot with the frequency axis. The frequency \(f_m\) was adjusted.
to provide the best fit of the volumetric flow curve obtained in the experiment to the output of the 4-element model of the input impedance, excited with the experimental pressure\(^{[1]}\).

**Fig.5.** Modulus and phase of the arterial input impedance of the carotid bifurcation models investigated. Modulus is scaled in \(10^8\, \text{[kg m}^{-4}\text{s}^{-1}]\), phase in deg.

### 3 RESULTS

The modulus of the input impedance increased for all harmonics, as did the phase values in the stenosed model with respect to the normal one (Fig.5, 6), regardless of how the impedance was obtained. These phenomena were due to both the increased hydraulic resistance and inertance of the narrowing. However, there were some differences between these three results (Fig.6). In particular, there were discrepancies between the values of the moduli obtained for the line transmission model with respect to the experimental results and lumped model results – the minimum of the modulus plot was closer to the origin in the case of the line transmission model, indicating a lower resonance frequency (Fig.6). However, the phase plot for the line transmission model was significantly closer to the experimental one than that obtained for the lumped model.

The values of serial resistances and sum of serial and peripheral resistances (modulus of the zeroth impedance component) of the 4-element impedance model increased for the stenosed bifurcation model (Table 3). The values of the compliance of the 4-element model fell for the stenosed model, possibly due to the change of the divider formed by the serial resistance of the stenosis and the peripheral resistance (Table 3). The values of the Pearson \(R^2\) coefficient, a measure of fit of the experimental flow curve and the response of the lumped electrical equivalent circuit excited with the experimental pressure, decreased for the stenosed model.

**Table 3.**

Values of parameters of the 4-element model of the arterial input impedance of the bifurcation models. Notation: \(R_0\) – serial resistance; \(R_p\) – peripheral resistance; \(C\) – compliance; \(L\) – inertance, \(R^2\) – Pearson coefficient. Viscosity 5.33mPas. Load compliance of the ICA branch \(1.054 \times 10^{-10} \, [\text{kg}^{-1}\text{m}^4\text{s}^2]\), of the ECA branch : \(1.200 \times 10^{-10} \, [\text{kg}^{-1}\text{m}^4\text{s}^2]\).
4 DISCUSSION AND CONCLUSIONS

The behaviour of the input impedance of the bifurcation model as a function of the
stenosis severity was generally coherent with the results of computation of the input impedance of the both electrical equivalent circuits of the hydraulic stand, especially for the normal model. Particularly interesting is that the phase plots from experiments and for the transmission line circuit were very close in the case of the normal model and in the case of the stenosed model for the first 3 harmonics differed slightly. It is to be noted, that the experimental results for higher harmonics might have been affected by the noise present mainly in the volume flow curve. This may indicate that even a simplified transmission line model is more appropriate than the lumped model.

The discrepancies observed between results for the two equivalent circuits may be due to the different nature of both (lumped – transmission line), as well as to the simplifications made in the analysis of the line transmission circuit. In particular the number of parts considered to have the transmission line nature was limited, the geometry of the stenosed model was significantly simplified when adopting the diameters of individual parts and no conduit tapering was taken into account.

The severe stenosis resulted in discrepancies between the impedance phase plots obtained for the lumped electrical equivalent circuit of the hydraulic stand and experimental results. This effect was manifested also as the decrease of $R^2$ value when comparing the experimental flow curve with the response of this circuit to the experimental pressure. This may be due to a factor not taken into account by the electrical equivalent circuit of the hydraulic stand. Possible candidate - the wave reflection at the stenosis site - demands further investigation.

The investigated method of measurement and analysis of the arterial input impedance may be considered an efficient tool for detection of the stenosis of the carotid bifurcation model and may be used for the assessment of the properties of the carotid arteries.

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