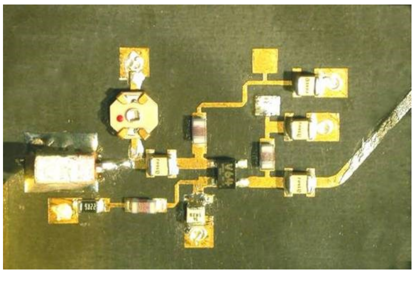
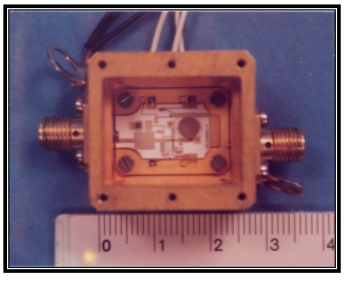
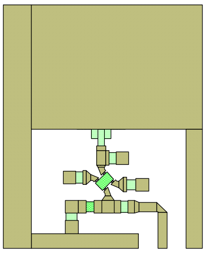
Negative Resistance Oscillators



Ceramic Oscillator (CRO)



Dielectric Oscillator (DRO)



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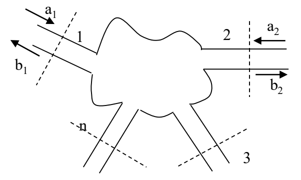


a_i = incident waves
b_i = reflected waves

$V_i = a_i + b_i$

$I_i = (a_i - b_i)/Z_{0i}$

Voltage waves

$$\mathbf{a}_i = \frac{\mathbf{V}_i + \mathbf{I}_i Z_{0i}}{2} \qquad \mathbf{b}_i = \frac{\mathbf{V}_i - \mathbf{I}_i Z_{0i}}{2}$$

Z_{0i} = Transmission Line Characteristic Impedance
Tipically $Z_{0i} = 50\ \Omega$

Why 50 Ohms?

Why was 50 ohms selected as the reference impedance of radar and radio equipment? There are many misconceptions surrounding this bit of history. "The first coaxial dimensions just happened to define the 50 ohm reference impedance." "Fifty ohms matched up well with the antennas in use 60 years ago." In actuality, the 50 ohm reference impedance was selected from a trade-off between the lowest loss and maximum power-handling dimension for an air line coaxial cable. The optimum ratio of the outer conductor to inner conductor, for minimum attenuation in a coaxial structure with air as the dielectric, is 3.6. This corresponds to an impedance, Zo, of 77 ohms.[5] Although this yields the best performance from a loss standpoint, it does not provide the maximum peak power handling before dielectric breakdown occurs. The best power performance is achieved when the ratio of the outer conductor to inner conductor is 1.65. This corresponds to a Zo of 30 ohms.[5] The geometric mean of 77 ohms and 30 ohms is approximately 50 ohms [Eq. (6.41)]; thus, the 50 ohm standard is a compromise between best attenuation performance and maximum peak power handling in the coaxial cable.

$$\boxed{50 \approx \sqrt{30 * 77}} \tag{6.41}$$

Power waves

$$\mathbf{V}_i = \sqrt{Z_{0i}}\left(\mathbf{a}_i + \mathbf{b}_i\right) \qquad \mathbf{I}_i = \frac{1}{\sqrt{Z_{0i}}}\left(\mathbf{a}_i - \mathbf{b}_i\right)$$

$$\mathbf{a}_i = \frac{1}{2}\left(\frac{\mathbf{V}_i}{\sqrt{Z_{0i}}} + \mathbf{I}_i\sqrt{Z_{0i}}\right) \qquad \mathbf{b}_i = \frac{1}{2}\left(\frac{\mathbf{V}_i}{\sqrt{Z_{0i}}} - \mathbf{I}_i\sqrt{Z_{0i}}\right)$$

Scattering matrix for a N-Port Network

For linear networks holds the **superposition principle**

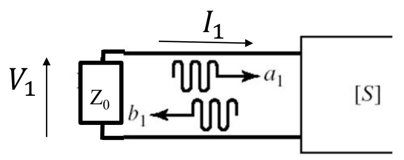
$$\begin{aligned}
\mathbf{b}_1 &= S_{11}\mathbf{a}_1 + S_{12}\mathbf{a}_2 + \ldots + S_{1n}\mathbf{a}_n \\
\mathbf{b}_2 &= S_{21}\mathbf{a}_1 + S_{22}\mathbf{a}_2 + \ldots + S_{2n}\mathbf{a}_n \\
\ldots & \quad \ldots\ldots\ldots \ \ldots\ldots\ldots \ \ldots\ldots\ldots \ \ldots\ldots \\
\mathbf{b}_n &= S_{n1}\mathbf{a}_1 + S_{n2}\mathbf{a}_2 + \ldots + S_{nn}\mathbf{a}_n
\end{aligned}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ .. \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \ldots.. & S_{1n} \\ S_{21} & S_{22} & \ldots.. & S_{2n} \\ \ldots. & \ldots. & \ldots.. & \ldots. \\ S_{n1} & S_{n2} & \ldots\ldots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \\ a_n \end{bmatrix}$$

[**b**] = [S] [**a**]

$$S_{ij} = \left(\frac{\mathbf{b}_i}{\mathbf{a}_j}\right)_{a_k = 0 \text{ con } k \neq j}$$

Matched Load



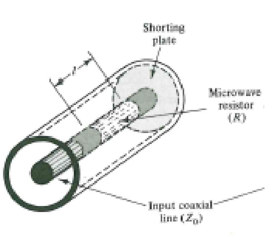
$$V_1 = -Z_0 I_1$$

$$a_1 = \frac{V_1 + Z_0 I_1}{2} = 0$$

When a network is closed on an matched load (50 Ω) the wave reflected by the load and incident on the port (a_1) is equal to zero

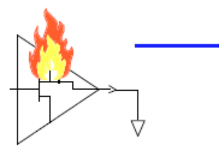
Why scattering parameters ?

- Scattering parameters can be measured at microwave frequencies as they require the network to be closed on 50 Ohm and this kind of loads are "relatively" easy to realize



Why scattering parameters ?

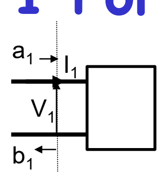
- At microwave frequencies, voltages and currents are hard to measure and the open and short circuits necessary to measure impedance and admittance parameters can cause the destruction of active devices



- The scattering parameters are defined for all two-port networks while the admittance and impedance parameters are not defined in some cases. For example, for a series impedance the impedance parameters are not defined because the currents at the two ports are not independent

Scattering matrix for a 1 Port Network

E = Incoming
I = Incident
R = Reflected



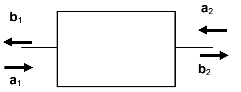
$$P_1 = P_E = \frac{1}{2}\mathrm{Re}\left(V_1 I_1^*\right) = \frac{1}{2}\mathrm{Re}\left\{\left(a_1 + b_1\right)\left(\frac{a_1 - b_1}{Z_0}\right)^*\right\} = \frac{1}{2Z_0}|a_1|^2 - \frac{1}{2Z_0}|b_1|^2$$

$$P_1 = P_E = P_I - P_R = P_I\left(1 - \frac{P_R}{P_I}\right) = P_I\left(1 - \frac{|b_1|^2}{|a_1|^2}\right)$$

$$\boxed{|S|^2 = |\Gamma_I|^2 = \frac{|b_1|^2}{|a_1|^2} = \frac{P_R}{P_I}}$$

the square modules of the scattering Parameters are ratios between powers

Scattering matrix for a 2 Port Network



$$P_2 = \frac{1}{2}\mathrm{Re}\left(V_2(-I_2)^*\right) = \frac{1}{2}\mathrm{Re}\left\{(a_2 + b_2)\left(\frac{b_2 - a_2}{Z_0}\right)^*\right\} = -\frac{1}{2Z_0}|a_2|^2 + \frac{1}{2Z_0}|b_2|^2$$

if $a_2 = 0 \longrightarrow P_2 = P_O = \frac{1}{2Z_0}|b_2|^2 = \frac{1}{2Z_0}|S_{21}|^2|a_1|^2 = P_I|S_{21}|^2 \longrightarrow \boxed{|S_{21}|^2 = \frac{P_O}{P_I}}$

if $a_2 = 0 \longrightarrow |\Gamma_I| = |S_{11}| \longrightarrow \boxed{|S_{11}|^2 = \frac{P_R}{P_I}}$ the square modules of the scattering Parameters are ratios between powers

Reciprocal two port network

(Absence of saturated ferrites or controlled generators in the component under test)

$[Z] = [Z]^T \longrightarrow \boxed{[S] = [S]^T}$

always true if [S] is defined starting from power waves

if [S] is defined starting from voltage waves, this is true if all the Z_{0i} are equal

$\boxed{S_{21} = S_{12}}$

Symmetrical two port network

A two port network is symmetrical if its input impedance is equal to its output impedance

Symmetrical networks are also physically symmetrical

$$S_{11} = S_{22}$$

Lossless two port network

$$[S]^{T*}[S] = [1] \qquad [S] = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \qquad [S^T] = \begin{bmatrix} s_{11} & s_{21} \\ s_{12} & s_{22} \end{bmatrix}$$

$$|s_{11}|^2 + |s_{12}|^2 = 1 \qquad |s_{21}|^2 + |s_{22}|^2 = 1$$

$$|s_{11}|\,|s_{21}|e^{j(\vartheta_{11}-\vartheta_{21})} + |s_{12}|\,|s_{22}|e^{j(\vartheta_{12}-\vartheta_{22})} = 0 \qquad \left\{ \begin{array}{l} |s_{11}|\,|s_{21}| = |s_{12}|\,|s_{22}| \\ \vartheta_{11} - \vartheta_{21} = \pi + \vartheta_{12} - \vartheta_{22} \end{array} \right.$$

$$|S_{21}| = |S_{12}| \qquad |S_{11}| = |S_{22}| \qquad |s_{12}| = \sqrt{1 - |s_{11}|^2}$$

Flow Graph Theory

Given a network with incident and reflected waves defined at the ports

a **node** (symbol ·) is a graphical symbol associated with each of these waves

A **branch** (symbol •-->--•) is an oriented line which connects two nodes. This line is oriented in the direction of the power flow

Each branch is linked to a **value** which gives the multiplicative factor that correlates the two waves at the ends of the branch (nodes)

A Path is a continuous set of branches all equally oriented that touch the single nodes only once. The value of the path is equal to the product of the values of the single branches

Loops of the 1st order are closed paths formed by branches all oriented in the same direction that touch the nodes only once. The value of the loop is equal to the product of the values of the single branches

Loops of the Second order are those formed by two loops of the 1st order with no node in common; the value of the 2nd order loop is equal to the product of the values of the two loops of the 1st order

Similarly for subsequent orders

Mason formula

$$T_{12} = \frac{P_1\left[1-\Sigma^{(1)}L(1)+\Sigma^{(1)}L(2)-\Sigma^{(1)}L(3)+\ldots\right]+P_2\left[1-\Sigma^{(2)}L(1)+\Sigma^{(2)}L(2)-\ldots\right]+P_3[\ldots]}{1-\Sigma L(1)+\Sigma L(2)-\Sigma L(3)+\ldots}$$

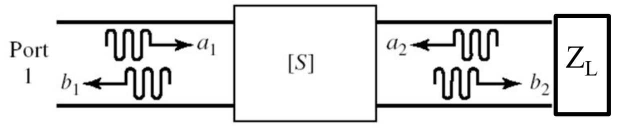
with

P_i = value of the i-th path between the two nodes under examination

$\Sigma L(i)$ = sum of all the possible loops of the i-th order

$\Sigma^{(k)}L(i)$ = sum of all possible loops of the i-th order without points in common with the k-th path

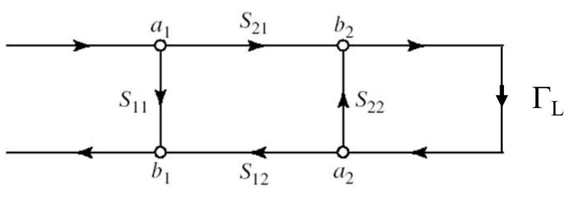
Example



$$b_1 = S_{11}a_1 + S_{21}a_2$$

$$\Gamma_L = \frac{a_2}{b_2} \qquad \Gamma_{IN} = \frac{b_1}{a_1}$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$



Paths

Loop

$P_1 = S_{11}$ $\qquad$ $P_2 = S_{21}\Gamma_L S_{12}$ $\qquad$ $L_1 = \Gamma_L S_{22}$

$$\Gamma_{IN} = \frac{S_{11}(1 - \Gamma_L S_{22}) + S_{21}\Gamma_L S_{12}}{1 - \Gamma_L S_{22}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - \Gamma_L S_{22}}$$

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - \Gamma_L S_{22}}$$

Negative Resistance Oscillator Theory

Oscillator structure

Oscillators convert energy from continuous to alternating, and are generally constituted by the three following subsystems:

- A non-linear active component (Z_D)
- A resonant structure that fixes the frequency of oscillation
- A transition for transferring the signal to the load (Z_C)

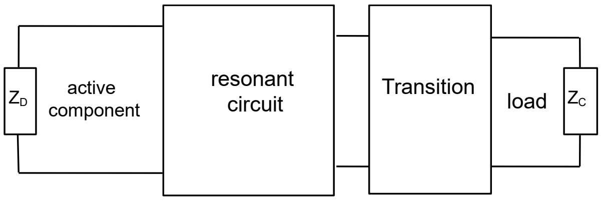


Figure 1

The active element of an oscillator can be seen as a negative resistance component

When two-terminal devices, such as Gunn or IMPATT diodes, are used the negative resistance behavior is achieved simply by polarizing the diode. For three-terminal devices, such as transistors, an appropriate feedback network must be added to achieve a negative resistance behavior

When fixed frequency performances are required, the **resonant structure** is generally constituted by dielectric or ceramic resonator. When tunable oscillators are required, YIG spheres or ceramic resonators tuned with varactor diodes can be used

The transition connects the source to the load, and can be used to improve some oscillator performance

Oscillator scheme

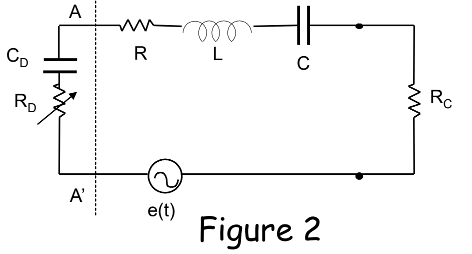


Figure 2

In this circuit **R_D and C_D** model the active element (R_D is a non linear resistance depending on the circuit current magnitude)

The R, L, C network models the resonator around its resonance frequency and it also takes into account parasitic present in the circuit

R_C represents the load, and the generator e(t) models the noise sources present in the circuit

Oscillator theory

This circuit can be studied in the complex frequency domain $s = \alpha + j\omega$ (neglecting the noise generator). In this case, we consider the complex current $I(s)$, and we place $R_T = R + R_C + R_{DL}$ (where R_{DL} is a linear approximation of R_D). Applying the kirchhoff law to the network we obtain:

$$[R_T + sL + 1/(sC_T)]\, I(s) = 0$$

This equation admits non-zero solutions only if the part between square brackets is null, and therefore:

$$s^2 LC_T + sC_T R_T + 1 = 0$$

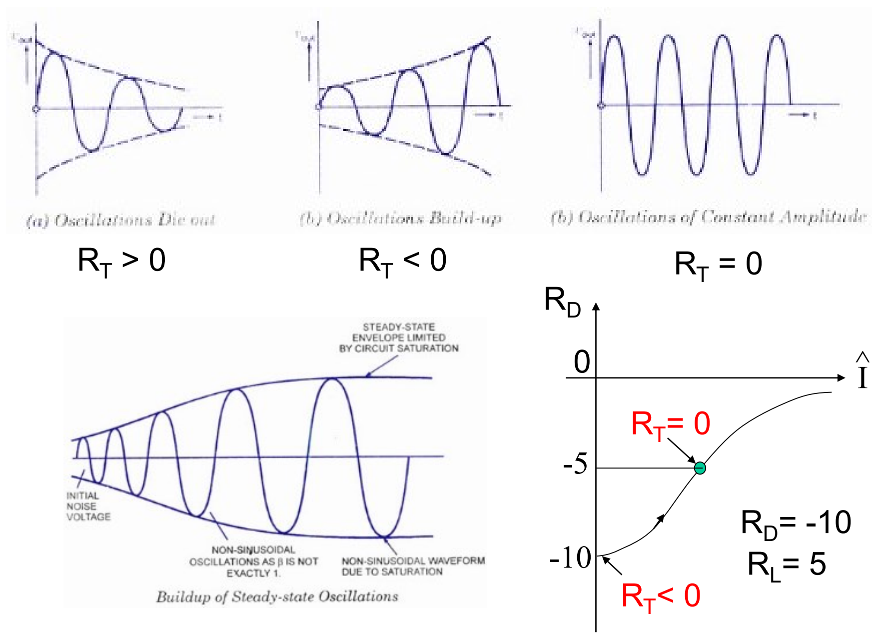
which admits two solutions:

$$s_{1,2} = \frac{R_T}{2L} \pm j\sqrt{\frac{1}{LC_T} - \left(\frac{R_T}{2L}\right)^2} = \alpha \pm j\widehat{\omega}$$

where $\widehat{\omega}$ is called the natural oscillation pulsation (pulsation = angular frequency)

The circuit current evolves over time as:

$$I(t) = \mathrm{Im}\left\{I(s)e^{st}\right\} = \hat{I}\, e^{-\frac{R_T}{2L}t}\, \mathrm{sen}(\hat{\omega}t + \varphi)$$



Steady state condition in terms of impedance

If $R_T = 0$, there are constant amplitude oscillations and the oscillation pulsation (resonance pulsation) is given by

$$\omega_0^2 = 1 / LC_T$$

from which it follows

$$\omega_0 L - 1/\omega_0 C_T = 0$$

or

$$X_L + X_C = X_T = 0$$

In conclusion we have:

$$R_T = 0$$

$$X_T = 0$$

That is:

$$Z_T = 0$$

These are the conditions for steady-state oscillations in terms of impedance

Steady state condition in terms of impedances at a section

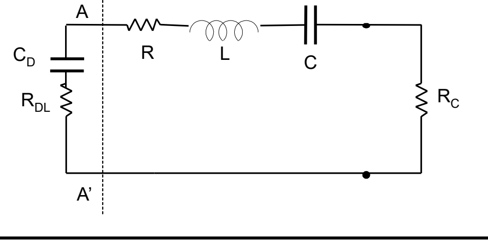
The steady-state conditions can also be applied by looking at the left and right of any circuit section

Therefore, if we consider the section AA' of the circuit scheme, indicating with Z_D the impedance (with a negative real part) of the active device and with Z_L the impedance of the whole passive circuit (resonator and load) we have for the total impedance Z_T :

$$Z_T = Z_D + Z_L = 0$$

$$R_T = R_D + R_L = 0$$

$$X_T = X_D + X_L = 0$$



Steady-state condition in terms of reflection coefficients at a section

The steady state condition can also be written according to the reflection coefficients Γ_D and Γ_L that are seen looking towards the active device and towards the load. In particular, we have:

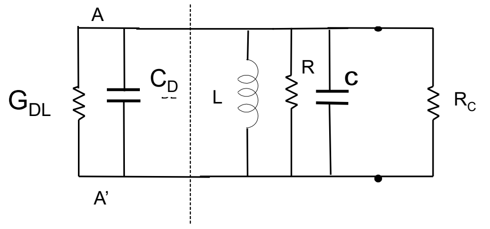
$$\Gamma_D \Gamma_L = \frac{Z_D - Z_0}{Z_D + Z_0} \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_D Z_L - Z_D Z_0 - Z_L Z_0 + Z_0^2}{Z_D Z_L + Z_D Z_0 + Z_L Z_0 + Z_0^2} = 1$$

That becomes:

$$|\Gamma_D \Gamma_L| = 1$$

$$\angle\Gamma_D + \angle\Gamma_L = 2n\pi$$

Steady-state condition in terms of admittances at a section



It should be noted that if instead of the previous series circuit we study a parallel configuration constituted by a parallel resonant circuit and an active device modeled by a G_{DL} in parallel with C_D, the parallel resonance condition will be reached:

$$Y_T = Y_D + Y_L = 0$$

$$G_T = G_D + G_L = 0$$

$$B_T = B_D + B_L = 0$$

For the oscillator design these conditions can be used, instead of the previous. **The choice depends on the behavior of the resonant circuit that may be series or parallel**

Oscillation startup condition

For the series resonant model, in order to startup the oscillations, it must be:

$$R_T < 0$$
$$X_T = 0$$

In fact, in this case the exponential coefficient of the current expression is positive and oscillations increase over time

For the parallel resonant model, must be:

$$G_T < 0$$
$$B_T = 0$$

With reference to the reflection coefficients, the $R_T < 0$ condition leads to the following results:

$$|\Gamma_D||\Gamma_L| > 1 \qquad (\text{if } R_D + R_L < 0, |R_D| < Z_0)$$

and

$$|\Gamma_D||\Gamma_L| < 1 \qquad (\text{if } R_D + R_L < 0, |R_D| > Z_0)$$

Similarly, from $G_T < 0$

(verify with examples:
$R_D = -10$, $R_L = 5$;
$R_D = -100$, $R_L = 90$)

Nyquist method for the oscillator design

Those seen so far are conditions for steady-state and startup of oscillations at a predetermined frequency expressed in terms of admittances impedances or reflection coefficients

The equations found can be used to design an oscillator, however, as seen before, these equations can be ambiguous in some cases

In addition, it is often useful to check what happens when the frequency changes to see if there are other oscillation frequencies in the circuit

Nyquist Theory

A rigorous and always correct way to verify the stability of a circuit is the Nyquist criterion

The Nyquist criterion is a graphical method that allows to determine the stability of a closed-loop system by evaluating its poles in the right half-plane

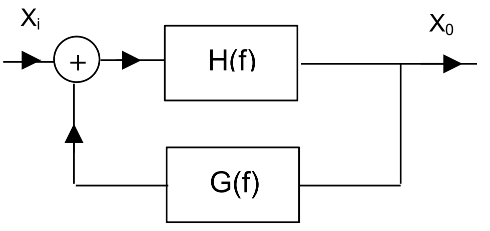


Figure 3

For the above system the input-output relation is:

$$X_0 = \frac{H(f)}{1 - G(f)H(f)} X_i \qquad (1)$$

where the G(f) H(f) product represents the open-chain transfer function

If we call:

P_{CL} (> 0) the number of poles with positive real part of the closed chain transfer function,

P_{OP} (> 0) the number of poles with positive real part of the open chain transfer function

N_T the number of turns that the open chain transfer function performs in the polar diagram around the critical point (1, j0), (considered positive clockwise and negative anticlockwise)

the closed-loop system is stable if and only if:

$$P_{CL} = P_{OP} + N_T = 0$$

If the open chain network is stable (P_{OP} = 0), it is sufficient that N_T = 0 (reduced criterion)

The Nyquist criterion requires the system modeled as a closed-loop system

This problem can be overcome by observing that for a generic oscillator consisting of an active part (Γ_D) and a passive load (Γ_L) it is possible to identify a transfer function similar to that of a closed-loop system

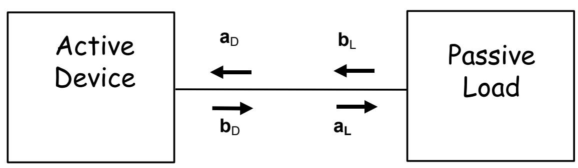
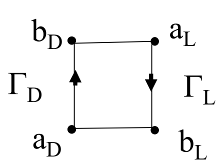


Figure 4

By using the flow graph method it results



$$\mathbf{a}_L = \frac{\Gamma_D(f)}{1-\Gamma_D(f)\Gamma_L(f)}\mathbf{a}_D \qquad (2)$$

The analogy between Eq (1) and (2) allows to study the stability of the system of Fig. 4 by means of the Nyquist criterion.
In this case the open chain transfer function is given by the product $\Gamma_D\,\Gamma_L$
This product can be easily evaluated with CAD systems available on the market through the use of an ideal circulator model inserted at the point of the circuit to be examined, according to the scheme of Fig. 5

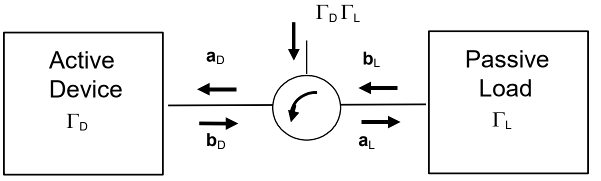


Figure 5

Since P_{OP} = 0 most of the time, the graph of $\Gamma_D \Gamma_L$ in the polar diagram allows to apply the reduced Nyquist criterion

$$N_T = 0$$

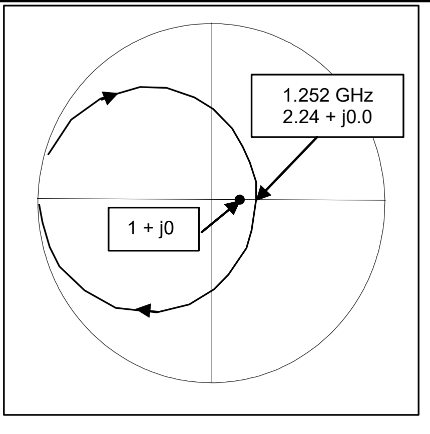


Figure 6

On the Nyquist plot it is also possible to evaluate the Oscillation frequency of the circuit that is equal to the frequency at which the plot crosses the positive axis (*)

(*) S. Pisa, E. Piuzzi, P. Tommasino, A. Trifiletti, A. Galli, G. Giolo, and A. Tafuto, "Desing, realization, and test of a 900 MHz ceramic oscillator," *Microwave Opt. Technol. Lett.*, vol. 49, no. 7, pp. 1713-1717, July 2007.

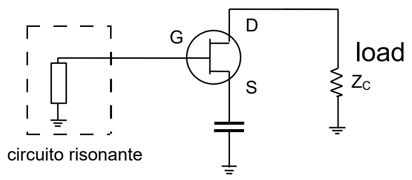
Negative Resistance Oscillators Realization

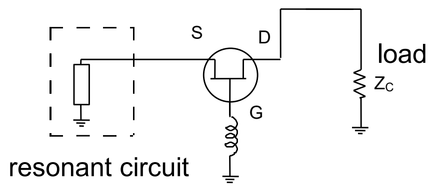
Transistor Oscillators

A peculiar characteristic of the microwave oscillators is that at these frequencies the parasitic elements of the transistors and in particular those that determine a feedback between the input and the output can not be neglected

In the oscillator design these elements are part of the feedback network, so the "external" feedback network can be considerably simplified compared to the low frequency case

For this reason, in the oscillator design stage at high frequencies, it is easier to think about the transistor with feedback as a "negative resistance" component





There are many ways to apply feedback and achieve negative resistance behavior

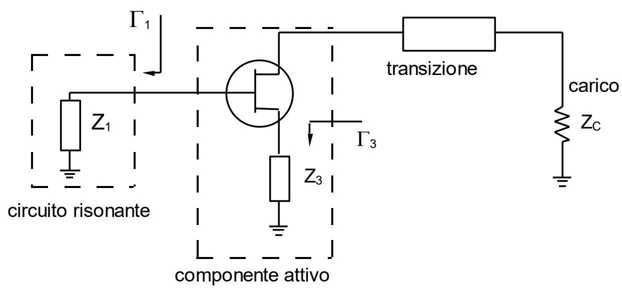
Two of the most used techniques are reported in the figures

a) Common Source with capacitive feedback

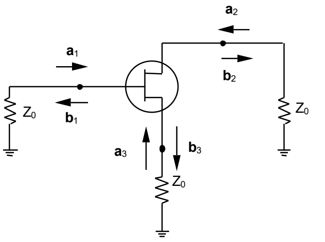
b) Common gate with inductive feedback

Capacitive Feedbak

The design of the capacitance can be approached in a simple way considering the transistor as a three-port component, or as a serie-serie connection of two networks



Transistor as a three port network



The access ports are defined between gate,
Drain, source and the ground.
The scattering matrix is the following:

$$\begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{pmatrix} \qquad (3)$$

For this device it holds: $I_1 + I_2 + I_3 = 0$

$$a_1 - b_1 + a_2 - b_2 + a_3 - b_3 = 0$$

Being $a_2 = a_3 = 0$ it results:

$$a_1 - S_{11}a_1 - S_{21}a_1 - S_{31}\, a_1 = 0$$

from witch:

$$S_{11} + S_{21} + S_{31} = 1$$

And in general:

$$\sum_{i=1}^{3} S_{ij} = 1 \qquad j = 1, 2, 3 \qquad (4)$$

If we place $V_1 = V_2 = V_3$, it results: $I_1 = I_2 = I_3 = 0$

$$a_1 + b_1 = a_2 + b_2 = a_3 + b_3$$

$$a_1 - b_1 = a_2 - b_2 = a_3 - b_3 = 0$$

from which it follows:

$$a_1 = a_2 = a_3 \text{ and } b_1 = b_2 = b_3.$$

$$a_1 = S_{11}a_1 + S_{12}a_1 + S_{13}\, a_1$$

$$S_{11} + S_{12} + S_{13} = 1$$

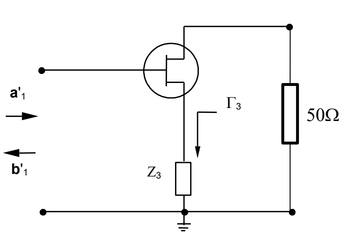
And in general:

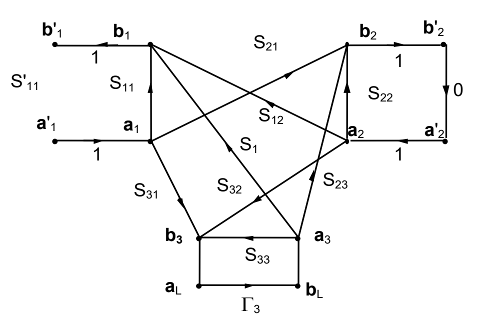
$$\sum_{j=1}^{3} S_{ij} = 1 \qquad i = 1,2,3 \qquad (5)$$

Combining (3), (4), (5) it is possible to achieve the 3 port parameters starting from the two port ones

Reactive Feedback

The scattering matrix of the transistor with capacitive feedback can be obtained starting from that of the Transistor as a 3-port device using the flow graph shown in the figure





For this network it results $S'_{11} = \left(\frac{b'_1}{a'_1}\right)_{a'_2=0}$

PATHS: $P_1 = S_{11}$ $P_2 = S_{31}\,\Gamma_3\,S_{13}$
LOOP: $L_1 = \Gamma_3\,S_{33}$

Solving the graph with the Mason's formula it results:

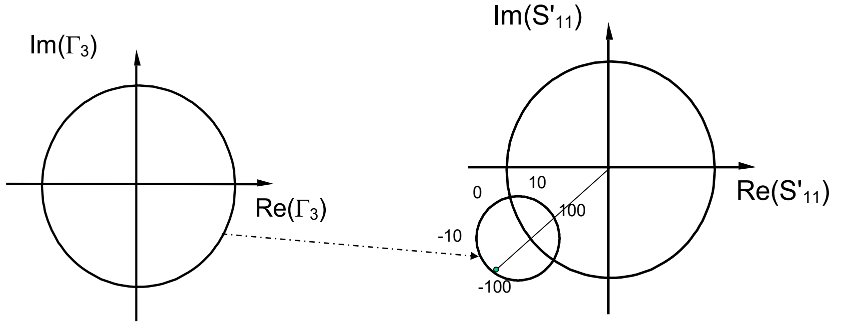
$$S'_{11} = \frac{S_{11}(1 - S_{33}\Gamma_3) + S_{31}\Gamma_3 S_{13}}{1 - S_{33}\Gamma_3} = S_{11} + \frac{S_{31}\Gamma_3 S_{13}}{1 - S_{33}\Gamma_3} \qquad (6)$$

The Z_3 impedance is designed in order to give, $|S'_{11}| > 1$ (negative input resistance)

Since Z_3 is a purely reactive load it results:

$$|\Gamma_3| = 1$$

For the Z_3 design it is therefore possible to draw in the plane of the input reflection coefficients (S'_{11}) the $|\Gamma_3| = 1$ locus



Eq (6) is a Conformal Transformation that transform Circumferences in the Γ_3 plane into circumferences in the S'_{11} plane

The figure shows that there are values of X_3 (Eg X_3 = -100) that make $|S'_{11}| > 1$

Series-Series Feedback

For the design of the capacity it is also possible to directly evaluate the scattering parameters of the transistor with the capacity in series with the source

In particular, from the scattering matrix of the transistor we can calculate the impedance matrix with the relation:

$$[Z_T] = Z_0\left([I] + [S_T]\right)\left([I] - [S_T]\right)^{-1}$$

The impedance matrix of the capacity is:

$$[Z_{c3}] = \frac{1}{j\omega C_3}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The connection between the transistor and the capacitance is of the series-series type, so the impedance matrix of the transistor with feedback is the sum of the impedance matrices of the transistor and of the inductance:

$$[Z_{TOT}] = [Z_T] + [Z_{LS}]$$

And the Total scattering matrix results:

$$[S_{TOT}] = \left([Z_{TOT}] + Z_0[I]\right)^{-1}\left([Z_{TOT}] - Z_0[I]\right)$$

So it is possible to set a simple program that changes C_3, evaluates S_{TOT} and choose the values of C_3 that give rise to the highest value of $| S_{TOT11} |$ (which in any case must be greater than 1)

Since many commercial CADs perform **parametric analysis**, another possible approach for dimensioning the Z_3 impedance is to use this option to graph the trend of the $|S_{11}|$ as a function of C_3 and then choose the C_3 value which maximizes $|S_{11}|$

Ceramic Resonator Oscillator (CRO)

Ceramic Resonator

Ceramic Resonators

Ceramic resonators (CR) are made with coaxial cable sections

One end of the center conductor (diameter d) is used as an input and it is connected to the outside with a pin

the other end can be opened or short-circuited with the outer armature (w side)

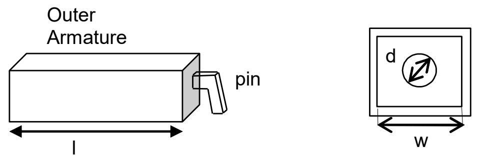


Figure 7

The outer armature has a square-section while the inner conductor has a circular section and in some cases is hollow

These features are used to make it easier the positioning and assembling of the resonator above planar circuits

Typically, the dielectric inside the cable is made of a ceramic material whose permittivity values are between 10 and 90

The characteristic impedance can vary between 10 and 30 Ω

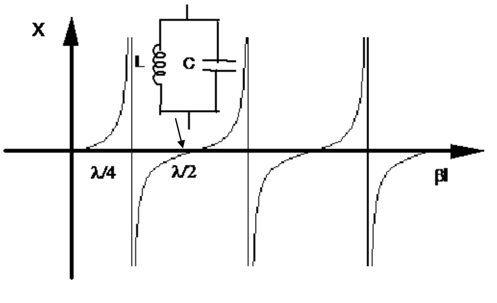
Since the ceramic materials have low losses these resonators have quality factors Q between 100 and 1000

Temperature coefficients lower than 10 ppm/°C are achieved

To understand the operation of ceramic resonators, we can observe that for a line of length "l" closed with a short circuit we have:

$$Z_{IN} = jZ_C \tan\beta l \qquad (7)$$

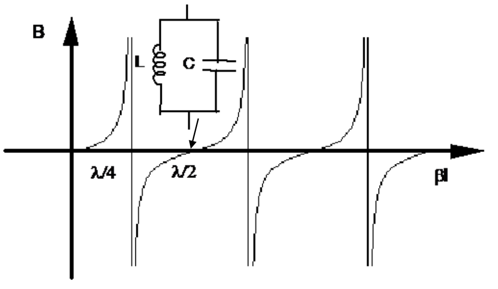
and then, if $l = \lambda / 4$ ($\beta l = \pi/2$) the line shows a parallel resonant circuit type behavior at the input port



Similarly, for a "l" length open line we have:

$$Y_{IN} = jY_C \tan \beta l \qquad (8)$$

and therefore, if $l = \lambda / 2$ ($\beta l = \pi$) the line still have a parallel resonant behavior



Although the name is ceramic resonators, these structures generally operate at frequencies below the resonance frequency (self resonant frequency - SRF) where they show an inductor-like behavior with high quality factors

The typical application of these resonators is in oscillators in which the resonator behaves like an inductance and resonates with the capacitive inputs of the transistors

For example, with reference to a short cable of length λ / 4, at frequencies much lower than the SRF, the cable behaves like an ideal inductance (the value of inductance does not depends on the frequency)

As the frequency increase, we approach the SRF and the shorted cable can be better modeled as a parallel RLC circuit

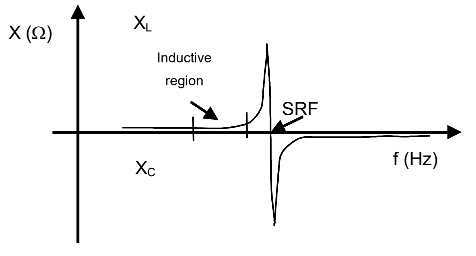


Figure 9

The length of the resonator can be chosen using equations (7) and (8)

However, the manufacturers of the resonators provide CADs through which, having set the desired inductance value and the resonance frequency SRF (typically between 10 and 20% greater than the working frequency), the parameters to be included in the simulation CAD are indicated together with the code of the component to be ordered

Finally, we can note that for the correct use of ceramic resonators it is important to consider also the additional inductance contribution due to the pin (see Fig. 7)

Experimentally it has been observed that a conductive wire adds a contribution of about 1 nH for each millimeter of length so that additional inductance contributions varying between 0.5 and 2 nH must be considered

CRO Design

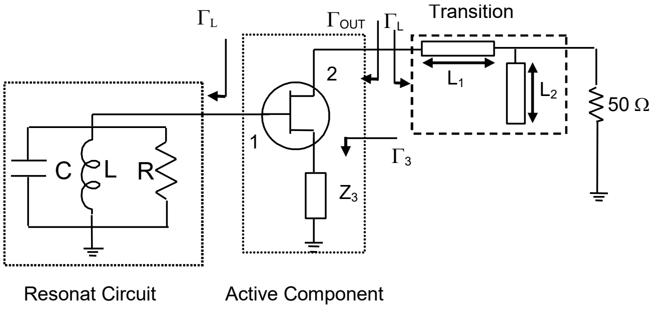


Figure 8

The oscillator design is performed eliminating the transition in Fig.8 and considering the transistor closed on a load equal to 50 Ω

The first step of the oscillator design consists in diemensioning the purely reactive element C_3 in order to obtain the maximum $|\Gamma_D|$ and in any case greater than 1 ($R_D < 0$)

This can be done by following the procedure described in the previous paragraph

The second step consists in dimensioning the input resonant circuit in order to obtain the desired oscillation frequency (SRF 10% or 20 % greater than oscillation frequency

Third step: Since the equivalent circuit of a dielectric resonator is a parallel RLC, the parallel resonance startup condition must be applied:

$$G_L + G_D < 0 \qquad (A)$$

$$B_L + B_D = 0 \qquad (B)$$

Given the low losses associated with the dielectric resonator, the (A) condition is almost always verified

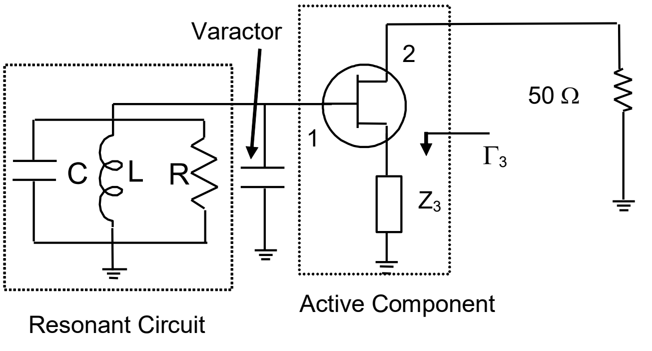
In order to satisfy (B) a dielectric resonator must be selected, for which, at the required resonance frequency it results:

$$B_L = -B_D$$

Since the input behavior of a transistor with capacitive feedback is typically capacitive the above condition is verified by operating the ceramic resonator in the inductive region to the left of the first parallel resonance

If $B_L + B_D = 0$ is not satisfied at the project frequency, a capacitor (or varactor) can be added in parallel to the resonant circuit

The varactor can be also used to realize a voltage controlled oscillator (VCO)



In order to verify the correctness of the design it is useful to evaluate, through the Nyquist plot, the intercept with the abscissa positive axis and then check if oscillations can be established in the circuit and their frequency

Moreover, with the Nyquist plot it is also possible to see if the circuit has spurious oscillations at different frequencies from the desired one

If the analysis with the Nyquist plot gives positive results, the realized device is able to oscillate on a load equal to 50 Ω

With these resonators, oscillators ranging from 200 MHz up to 5 or 6 GHz can be realized. It is not possible to operate at higher frequencies because the mechanical dimensions of the resonator become too small

Using CAD in which analysis techniques are implemented for non-linear structures it is then possible to evaluate the oscillator performance in terms of effective resonance frequency, output power, harmonics and phase noise

Using these CAD it is also possible to dimension the output transition network in order to optimize the oscillator performance in terms of phase noise, output power, harmonics, etc.

Dielectric Resonator Oscillator (DRO)

Dielectric Resonator

Dielectric Resonators characteristics

Dielectric resonators (DR) are made with ceramic materials with dielectric constants varying between 30 and 100

The dimensions of these resonators, for a given operating frequency, compared to those of resonators made with empty metal cavities, are smaller than a factor equal to the square root of ε_r

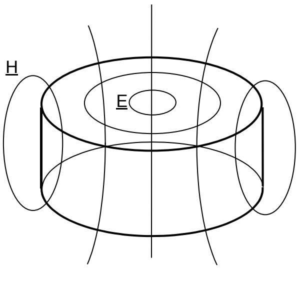
Fig. 9 shows the geometry and the force lines of the electric and magnetic field for the $TE_{01\delta}$ mode of a cylindrical resonator

Remember that the index 0 indicates the order of the Bessel function, the index 1 indicates the order of the root, the real index δ indicates that the spatial variation of the field along the resonator axis is a non-integer multiple of a half wavelength. More details can be found in: D. Kajfez, P. Guillon: Dielectric resonator, A. House, 1986

As can be seen the force lines of the electric field are concentric circumferences around the cylinder axis while those of the magnetic field are ellipsoids that lie in the meridian plane

The losses, and therefore the merit factor of the $TE_{01\delta}$ mode, are essentially connected to the losses in the dielectric and we obtain values of Q ranging from 1000 to 10000 at frequencies between 1 and 30 GHz

The temperature coefficient of the resonant frequency ($\Delta f/\Delta T$) includes the combined effect of the temperature coefficient of the permittivity and of the thermal expansion of the dielectric. Typical values for this coefficient vary between -9 and +9 ppm / ° C (ppm = parts per million)



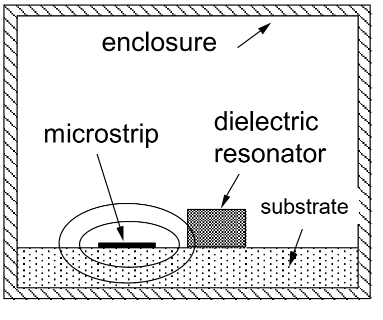


Figure 9

Dielectric Resonators model

The assembly of the DR is generally carried out as in Fig. 9. The resonator is placed over the substrate of the microstrip: in this way the magnetic field of the microstrip is able to excite the $TE_{01\delta}$ mode

The lateral distance between the resonator and the line determines the degree of coupling between the two

The equivalent circuit of a resonator coupled to the microstrip line (Fig. 10.a) is shown in Fig. 10.b. In the figure, L_s, C_s and R_s model the resonator around the resonance frequency, while L_1 and L_m model the magnetic coupling.

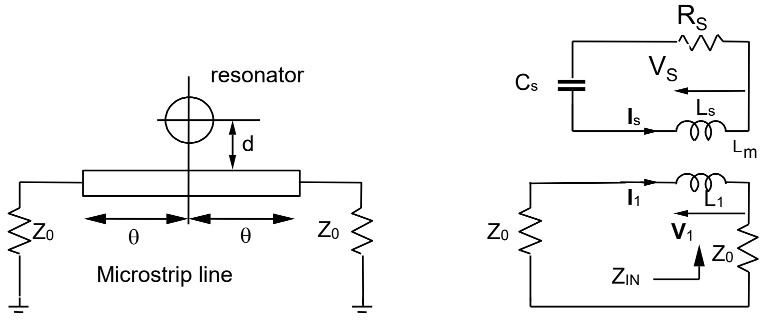


Figure 10

The circuit equations are:

$$\text{(A)}\quad \begin{aligned} \mathbf{V}_1 &= j\omega L_1 \mathbf{I}_1 + j\omega L_m \mathbf{I}_s \\ \mathbf{V}_s &= j\omega L_s \mathbf{I}_s + j\omega L_m \mathbf{I}_1 \end{aligned} \qquad Z_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = j\omega L_1 + j\omega L_m \frac{\mathbf{I}_s}{\mathbf{I}_1}$$

From the loop in the second branch of the transformer it results:

$$V_S + I_S R_S + I_S / j\omega C_S = 0$$

Combining this equation with (A) it results

$$I_S R_S + I_S / j\omega C_S + j\omega L_S I_S + j\omega L_m I_1 = 0$$

$$\frac{I_S}{I_1} = \frac{-j\omega L_m}{R_S + j\omega L_s + \frac{1}{j\omega C_S}}$$

$$Z_1 = j\omega L_1 + \frac{\omega^2 L_m^2}{R_S + j\omega L_s + \frac{1}{j\omega C_S}}$$

Around the resonance frequency it results:

$\omega = \omega_0 + \Delta\omega$ where: $\omega_0^2 C_s L_s = 1$

For the Z_1 denominator it results:

$$j(\omega_0 + \Delta\omega)L_s + \frac{1}{j(\omega_0 + \Delta\omega)C_s} =$$

$$j\omega_0 L_s + j\Delta\omega L_s + \frac{-j(\omega_0 - \Delta\omega)}{(\omega_0 + \Delta\omega)(\omega_0 - \Delta\omega)C_s} =$$

$$j\omega_0 L_s + j\Delta\omega L_s - \frac{j}{\omega_0 C_s} + \frac{j\Delta\omega}{\omega_0^2 C_s} = j2\Delta\omega L_s$$

$$Z_1 = \frac{\omega_0^2 L_m^2}{R_S + j2\Delta\omega L_s} = \frac{\frac{\omega_0^2 L_m^2}{R_S}}{1+\frac{j2\Delta\omega L_S\,\omega_0}{R_S\,\omega_0}} = \frac{\frac{\omega_0^2 L_m^2}{R_S}}{1+\frac{j2Q_{US}\Delta\omega}{\omega_0}} = \frac{R_P}{1+\frac{j2Q_{US}\Delta\omega}{\omega_0}}$$

For this structure the series (Q_{US}) and parallel (Q_{UP}) unloaded Q are equal and therefore:

$$Q_{US} = \frac{\omega_0 L_s}{R_S} = Q_{UP} = \omega_0 C_P R_P \qquad R_P = \frac{\omega_0^2 L_m^2}{R_S}$$

Moreover the two circuits have the same resonance frequency and so:

$$C_s L_s = C_P L_P$$

The external and loaded Q are:

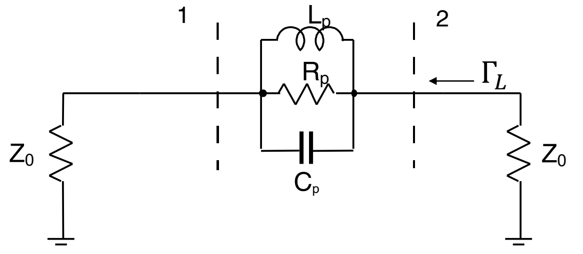
$$Q_{EP} = \frac{2Z_0}{\omega_0 L_P} \qquad Q_{LP} = \frac{2Z_0 // R_P}{\omega_0 L_P}$$

And it results:

$$\frac{1}{Q_{LP}} = \frac{1}{Q_{EP}} + \frac{1}{Q_{UP}}$$

The coupling coefficient is: $\beta = \frac{Q_{UP}}{Q_{EP}} = \frac{R_P}{2Z_0}$

In conclusion the following circuit can be drawn



where:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(Z_1 + Z_0) - Z_0}{(Z_1 + Z_0) + Z_0} = \frac{Z_1}{Z_1 + 2Z_0} =$$

$$\frac{\dfrac{R_P}{1 + \dfrac{jQ_U \Delta\omega}{\omega_0}}}{\dfrac{R_P}{1 + \dfrac{j2Q_U \Delta\omega}{\omega_0}} + 2Z_0} = \frac{R_P}{R_P + 2Z_0 + 2Z_0 \dfrac{j2Q_U \Delta\omega}{\omega_0}} = \frac{\beta}{1 + \beta \quad + \dfrac{j2Q_U \Delta\omega}{\omega_0}}$$

Thus the presence of the mutual inductance in the model has transformed a series resonant circuit into a parallel resonant circuit

Moreover, the reflection coefficient (Γ_1) depends on the coupling coefficient β which in turn depends on the distance between the resonator and the microstrip

DRO Design

Oscillators using dielectric resonators are commonly referred to as dielectric resonators (Dielectric Resonator Oscillator - DRO). A possible configuration for these oscillators is shown in the figure below

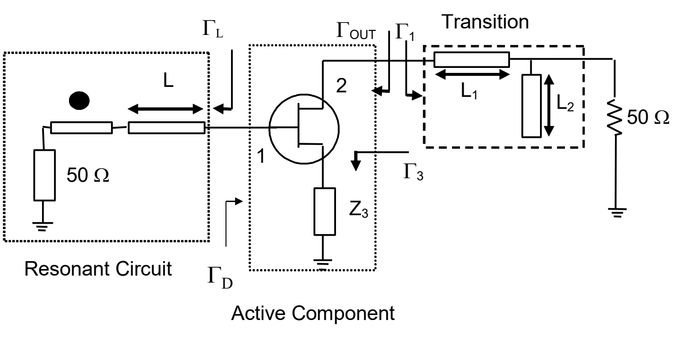


Figure 11

1st step: to apply the feedback to the transistor to achieve an input negative resistance

2nd step: to chose a resonator with the desired resonance frequency

3rd step: to determine the value of the reflection coefficient Γ_L in Fig. 11 to verify the condition for the startup of the oscillations. We use the startup condition in terms of reflection coefficients. Being the dielectric resonator modelled as a parallel RLC circuit and being, in practical cases, always verified the condition $Y_0 < |G_D|$ must be:

$$|\Gamma_L||\Gamma_D| > 1$$

$$\angle \Gamma_L + \angle S'_{11} = 0$$

For dimensioning $|\Gamma_L|$ and $\angle \Gamma_L$ two specific procedures can be used.

<u>As for the $|\Gamma_L|$</u> this depends on the type of DR used and on the distance between the DR and the microstrip line. This link has been previously evaluated and we obtain:

$$|\Gamma_L| = \left|\frac{\beta}{1+\beta \quad +\frac{j2Q_U\Delta\omega}{\omega_0}}\right| > \left|\frac{1}{\Gamma_D}\right|$$

There are various software on the market, often supplied by the same companies that produce dielectric resonators, through which it is possible to evaluate $|\Gamma_L|$ starting from the dimensions of the resonator, from its dielectric characteristics and from the distance between the resonator and the microstrip.

In particular, once chosen the resonator for the desired resonance frequency we can vary the distance between the resonator and the microstrip to find a value of $|\Gamma_L|$ that meets the previous equation

In order to satisfy the resonance condition for the phase, we must note that at the resonance the DR is purely resistive, and its phase is zero.

To satisfy the phase condition a line L can be inserted between the resonator and the active circuit (see Fig. 11)

with this choice the condition on the phase becomes:

$$-2\beta L + \angle \Gamma_D = 0$$

The length of the line must be chosen to satisfy this equation at the resonance frequency of the DRO

In order to verify the correctness of the dimensioning it is useful to carry out the analysis with the Nyquist plot previously described

By evaluating the intercept with the abscissa positive axis of the Nyquist plot, it is possible to detect if oscillation occur in the circuit and to establish their frequency

Moreover, it is also possible to see if the circuit has spurious oscillations at different frequencies from the planned one

If the Nyquist analysis gives positive results, the component thus realized is able to oscillate on a load equal to 50 Ω

Using CAD in which analysis techniques are implemented for non-linear structures it is then possible to evaluate the oscillator performance in terms of effective resonance frequency, output power, harmonics and phase noise