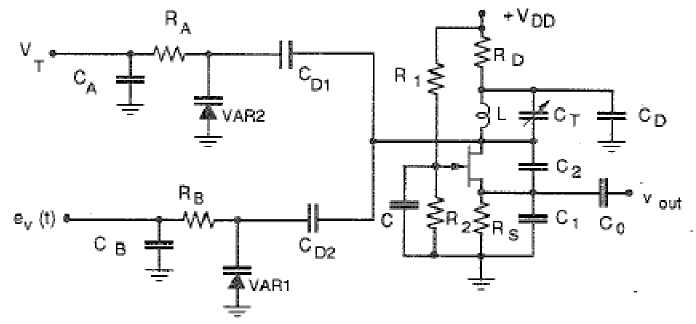
Voltage controlled oscillator (VCO)



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Oscillation frequency

$$Z = \frac{j\omega L}{1 - \omega^2 LC(V)} \cong j\omega L[1 + \omega^2 LC(V)]$$

$$\omega_0^2 = \frac{1}{\frac{C_1 C_2}{C_1 + C_2}[L + \omega_0^2 L^2 C(V)]} \cong \frac{1}{L\left[C_C + C(V)\right]} = \frac{1}{LC_T}$$

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VCO gain /1

$$2\cdot \mathrm{Log} f_0 = -\mathrm{Log}\left\{4\pi^2 L\left[C_C + C(V)\right]\right\}$$

$$\frac{\Delta f_0}{f_0} = \Delta \mathrm{Log} f_0 = -1/2\,\frac{4\pi^2 L \Delta C(V)}{4\pi^2 L\left[C_C + C(V)\right]} = -2\pi^2 L \Delta C(V) f_0^2$$

$$\Delta C(V) = -\frac{\Delta f_0}{2\pi^2 L f_0^3}$$

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VCO gain /2

$$C(V) = \frac{C_{j0}}{\sqrt{1 + V/V_{Bi}}} \qquad \Delta C(V) = -1/2\,\frac{1}{V_{Bi}}\,\frac{C_{j0}}{(1+V/V_{Bi})^{3/2}}\,\Delta V = \frac{-1/2\sqrt{V_{Bi}}\,C_{j0}}{(V_{Bi}+V)^{3/2}}\,\Delta V$$

$$k_V = \frac{\Delta f_0}{\Delta V} = \frac{\Delta f_0}{\Delta C}\cdot\frac{\Delta C}{\Delta V} = \pi^2 L f_0^3 \cdot \frac{\sqrt{V_{Bi}}\,C_{j0}}{(V_{Bi}+V)^{3/2}}$$

Poiché: $f_0 = \frac{1}{(2\pi)\sqrt{LC_T}}$

$$k_V(V) = \frac{\sqrt{V_{Bi}}\,C_{j0}}{8\pi\sqrt{L}[C_T(V_{Bi}+V)]^{3/2}} = \frac{\sqrt{V_{Bi}}\,C_{j0}}{8\pi\sqrt{L}[\left(C_c + \frac{C_{j0}}{\sqrt{1+V/V_{Bi}}}\right)(V_{Bi}+V)]^{3/2}}$$

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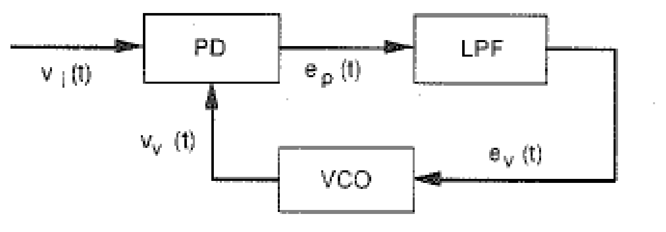
Phase-locked loop (PLL) /1

- PLL is a system that allows synchronization of local oscillator in the receiver to received data: the phase-locked oscillator can be used or received signal regeneration
- The system performs evaluation of the phase of received signal, and then synchronization of VCO phase by means of a DC feedback loop

Phase-locked loop (PLL) /2



- The overall system comprises a Phase Detector, a low-pass filter with pulse response w(t), and a VCO

Phase-locked loop (PLL) /3

- Under the hypothesis that the received signal $v_i(t)$ shows instantaneous phase $\varphi_i(t)$, and that at initial time instant it shows a phase difference $\varphi_{vi}(0)$ with respect to the signal at VCO output:

$$v_v(t) = A_v(t) \cdot \sin(\omega_v\, t + \varphi_{vi}(0))$$
$$v_i(t) = A_i(t) \cdot \sin(\omega_i\, t)$$

- K_p is the phase detector gain, and K_v the VCO gain

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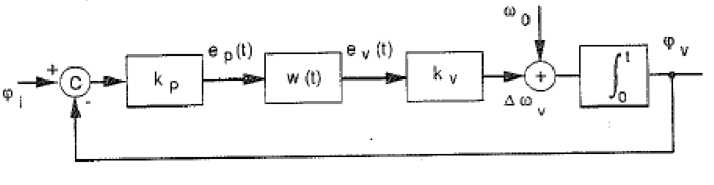
Phase-locked loop (PLL) /4

- The system can be represented by means of the equivalent scheme below:



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Phase-locked loop (PLL) /5

$e_p(t) = K_p \cdot [\varphi_i(t) - \varphi_v(t)]$ ⟺ $E_p(s) = K_p \cdot [\varphi_i(s) - \varphi_v(s)]$

$e_v(t) = e_p(t) * w(t)$ ⟺ $E_v(s) = E_p(s) \cdot W(s)$

$\omega_v(t) = \omega_0 + K_v \cdot e_v(t)$

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Phase-locked loop (PLL) /6

- In order to understand the PLL dynamic behavior, we suppose that at the initial time instant the system is in lock ($\omega_i = \omega_0$, $e_v = 0$).
- Moreover, we suppose a little deviation $\varphi_i(t)$ (i.e. a small-signal model can be used) with respect to the value φ_{i0} at lock:

$$\varphi_{iTOT}(t) = \varphi_{i0} + \varphi_i(t)$$
$$\varphi_{vTOT}(t) = \varphi_{v0} + \varphi_v(t) = \varphi_{i0} + \varphi_v(t)$$

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Phase-locked loop (PLL) /7

- From linear analysis, PLL transfer functions as a function of the loop gain T(s) are derived:
  - $T(s) = K_p \cdot K_v \cdot W(s) / s$
  - $\varphi_v(s) / \varphi_i(s) = T(s) / [1 + T(s)] = H(s)$
  - $\Delta\varphi(s) / \varphi_i(s) = [\varphi_i(s) - \varphi_v(s)] / \varphi_i(s) = 1 / [1 + T(s)]$
- If $K_p \cdot K_v \rightarrow \infty$ the VCO phase is locked to the received signal phase

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Phase-locked loop (PLL) /8

- PLL operation can be comprised by evaluating the response to a step input signal, starting from the lock condition. Both a phase step and a frequency step are considered:
- In case of phase step $\Delta\varphi_i$ we get:

$$\lim_{t \to \infty} \Delta\varphi(t) = \lim_{s \to 0} s \cdot \Delta\varphi(s) = \lim_{s \to 0} s \cdot \Delta\varphi_i/s \cdot 1 / [1 + T(s)] = 0$$

- In case of frequency step $\Delta\omega_i$ we get:

$$\lim_{t \to \infty} \Delta\varphi(t) = \lim_{s \to 0} s \cdot \Delta\varphi(s) = \lim_{s \to 0} s \cdot \Delta\omega_i/s^2 \cdot 1 / [1 + T(s)] = \Delta\omega_i / [K_p \cdot K_v \cdot W(0)]$$

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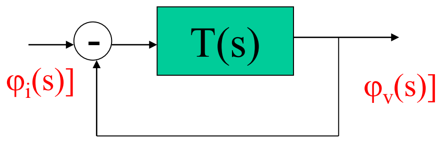
Phase-locked loop (PLL) /9

- The PLL behaves as a unitary feedback system with loop gain T(s).



- Loop stability can be checked by tracing the root locus of loop gain T(s), as a function of the filter transfer function W(s)

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Phase-locked loop (PLL) /10

- If W(s) = 1 (no filter), T(s) shows a single pole in the origin, and therefore no instability issues are found
- If W(s) = 1 / (1 + s R C) (R-C filter) T(s) shows a pole in the origin and a second pole s = -1 / (R C): an oscillation behavior is found in the time response, and the damping factor ξ decreases as the loop gain $K_p \cdot K_v$ increases:

$$\omega_n = \sqrt{\frac{K_p \cdot K_v}{R\,C}} \qquad \xi = \frac{1}{2 \cdot \sqrt{R \cdot C \cdot K_p \cdot K_v}}$$

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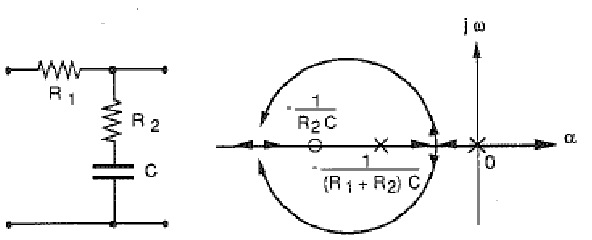
Phase-locked loop (PLL) /11

- In order to induce oscillation damping, a zero is added in the filter transfer function, so that the following root locus is found:



Phase-locked loop (PLL) /12

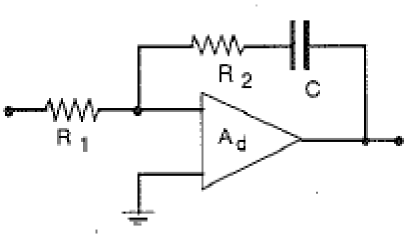
$$\omega_n = \sqrt{\frac{K_p \cdot K_v}{(R_1 + R_2)C}} \qquad \xi = \frac{1 + K_p \cdot K_v \cdot R_2 \cdot C}{2 \cdot \sqrt{(R_1 + R_2)C \cdot K_p \cdot K_v}}$$

- Loop gain expression is:

$$T(s) = \frac{K_p \cdot K_v}{s} \frac{1 + s \cdot R_2 \cdot C}{1 + s \cdot (R_1 + R_2)C}$$

Phase-locked loop (PLL) /13

- A more efficient filter topology can be considered exploiting an operational amplifier (active filter)



Phase-locked loop (PLL) /14

- In this case we have $T(s) = K_p \cdot K_v / s \cdot (1 + s R_2 C) / (s R_1 C)$.
- In particular, W(s) shows a pole in the origin, and the system allows phase lock even in presence of a frequency step

$$\omega_n = \sqrt{\frac{K_p \cdot K_v}{R_1 C}} \qquad \xi = \frac{\sqrt{(R_2{}^2 / R_1) C \cdot K_p \cdot K_v}}{2}$$

Phase-locked loop (PLL) /15

- The following expression for H(s) is found:

$$H(s) = \frac{2s\xi\omega_n + \omega_n^2}{s^2 + 2s\xi\omega_n + \omega_n^2}$$

- We'll see below the effect of PLL in presence of white phase noise

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Phase-locked loop (PLL) /16

- The Lock Range is a parameter which accounts for PLL capability to recover instantaneously (i.e. within a single period) the lock condition, in presence of phase noise
- The lock Time T_L shows inverse proportionality versus the natural radian frequency of poles ω_n. The lock range $\Delta\omega_L$ instead shows direct proportionality versus ω_n. Therefore, a more fast synchronization is obtained with a grater ω_n value (a greater loop bandwidth)
- However, a greater loop bandwidth produces more phase noise at the output

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Phase-locked loop (PLL) /17

- The Lock Range can be evaluated by supposing a stepwise input $\Delta\omega_i$: it corresponds to an input phase error $\Delta\varphi_i = \Delta\omega_i \cdot t$, and at PD output (if an analog multiplier is used) we find:

$$e_p(t) = K_p \cdot \cos(\Delta\omega_i\ t)$$

- At filter output ($W(j\omega) = |W(j\omega)| \cdot \exp(\varphi_W(j\omega))$ is filter response), the signal $e_v(t)$ is:

$$e_v(t) = K_p\ |W(j\Delta\omega_i)| \cdot \cos(\Delta\omega_i\ t + \varphi_W(\Delta\omega_i))$$

- And maximum radian frequency variation at VCO output is:

$$\Delta\omega_{MAX} = K_p\ K_v\ |W(j\ \Delta\omega_i)|$$

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Phase-locked loop (PLL) /18

- The condition to obtain PLL lock within a period, as a function of VCO tuning range and loop gain, is the following:

$$\Delta\omega_L = K_p\ K_v\ |W(j\ \Delta\omega_L)|$$

- If an active filter is consider, we have:

$$\Delta\omega_L = K_p\ K_v\ R_2 / R_1 = 2\ \xi\ \omega_n$$

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Noise sources in electronic devices /1

$$\overline{e_T^2} = 4kTBr_b$$

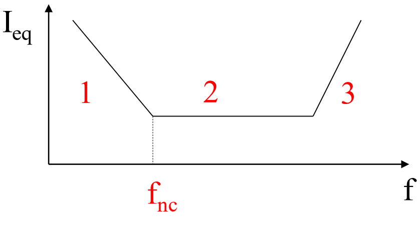
$$\overline{i_{Shot}^2} = 2q\overline{I_C}B$$

$$\overline{i_{Flicker}^2} = \int_B K_F \frac{\overline{I_B^{A_F}}}{f} df$$

- The presence of such noise sources in a 2-port network can be represented by means of a noisy equivalent model

Noise sources in electronic devices /2

- The power spectral density of current noise in a CE amplifier shows the behavior depicted below, in which three main regions can be seen:
  - 1. 1/f behavior due to Flicker noise (up to some KHz)
  - 2. Flat behavior due to white noise (Shot & Johnson)
  - 3. f^2 behavior due to transistor dominant pole



Phase noise in oscillators - 1

- Due to the presence o device noise sources, noise is added to both amplitude and phase of the oscillators:

$$v_{out}(t) = [\hat{V} + v(t)] \cdot \sin[\omega_0 t + \varphi(t)]$$

$$\omega(t) = \omega_0 + \frac{d\varphi(t)}{dt}$$

- Amplitude modulation can be cancelled by means of a limiter, while phase noise cannot be eliminated

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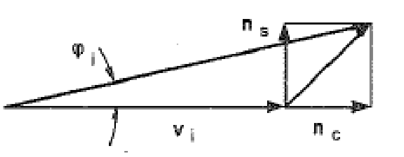
Phase noise in oscillators - 2



- Noise can be considered as a phasor that produces modification of the output signal phasor: both the amplitude (but it can be neglected) and the phase are affected. For noise power (i.e. noise variance) we get:

$$\varphi_i(t) = \mathrm{arctg}\left(\frac{n_s(t)}{\hat{V} + n_c(t)}\right) \cong \frac{n_s(t)}{\hat{V}} \quad \Rightarrow \quad \overline{\varphi_i^2}(t) = \frac{n \cdot B}{2S} = \frac{N}{2S}$$

- As it can be seen, power noise is inversely proportional to signal-to-noise ratio

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Phase noise in oscillators - 3

- The signal spectrum shows 2 side-bands with different power, together with the central frequency (i.e. the oscillation frequency f_0)
- If we evaluate the spectral component at f_m frequency of a signal composed of 2 side-bands B_1 (at $-f_m$) e B_2 (at f_m), and f_0

$$v(t) = A \cdot \cos(\omega_0 t) + B_1 \cdot \cos[(\omega_0 + \omega_m)t] + B_2 \cdot \cos[(\omega_0 - \omega_m)t]$$

- It can be considered as the superposition of an amplitude modulation and a low-index phase modulation

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Phase noise in oscillators - 4

- A symmetrical spectrum is produced by amplitude modulation

$$v(t) = A \cdot [1 + \cos(\omega_m t)] \cdot \cos(\omega_0 t) =$$
$$A \cdot \cos(\omega_0 t) + \frac{A}{2}\cos[(\omega_0 + \omega_m)t] + \frac{A}{2}\cos[(\omega_0 - \omega_m)t]$$

- If we suppose a cosine waveform with unitary amplitude and phase modulation with maximum phase deviation $\beta << 90°$:

$$v(t) = \cos[\omega_0 t + \beta \cdot \sin(\omega_m t)]$$

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Phase noise in oscillators - 5

- By using trigonometric transformations, we get:

$$v(t) = \cos(\omega_0 t) - \frac{\beta}{2}\cos[(\omega_0 - \omega_m)t] + \frac{\beta}{2}\cos[(\omega_0 + \omega_m)t]$$

- In the following slide, we can see the overall signal together with the 2 modulated signals

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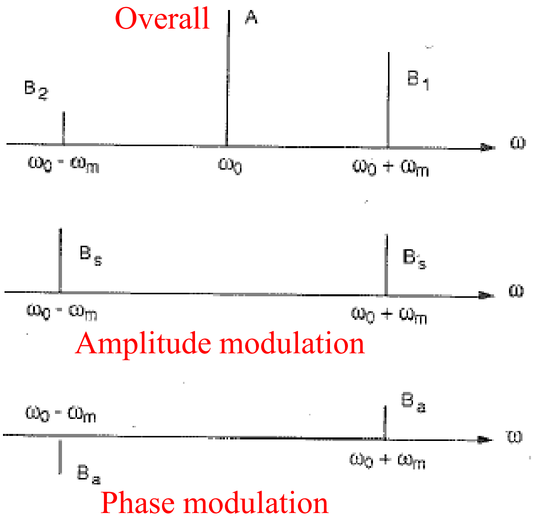
Phase noise in oscillators - 6

Overall



Amplitude modulation

Phase modulation

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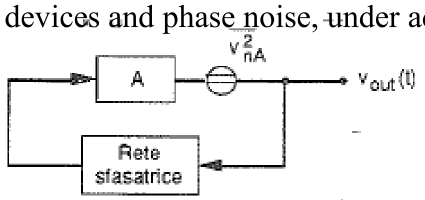
Phase noise in oscillators - 7

- The Figure of Merit used to account for phase noise performance is Single Sideband to Carrier Ratio SSCR(ω), the ratio between noise in a single sideband $\Delta\omega = 2\pi\cdot 1\text{Hz}$ placed at ω distance from ω_0, and the overall power:

$$SSCR(\omega) = 10 \cdot \log \frac{S_{VV}(\omega) \cdot \Delta\omega}{P_{tot}}$$

- Now, a link will be found between physical noise sources in electronic devices and phase noise, under additive noise hypothesis



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Phase noise in oscillators - 8

- Block A amplification is considered as real

- Noise is ‘filtered’ by a resonant network characterized with its frequency stability coefficient S_F:

$$\Delta\omega = \frac{\omega_0}{S_F}\Delta\varphi \quad \Rightarrow \quad S_{\varphi\varphi}(\omega) = \frac{S_{\omega\omega}(\omega)}{\omega^2} \propto \left(\frac{\omega_0}{S_F}\right)^2 \frac{N(\omega)}{\omega^2}$$

- Noise N(ω) of electronic devices produces transfer function phase variation Δφ

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Phase noise in oscillators - 9

- Phase noise spectrum can be evaluated from frequency dependence on noise spectrum
- Around ω_0 Flicker noise is the main contribution and the spectrum is:

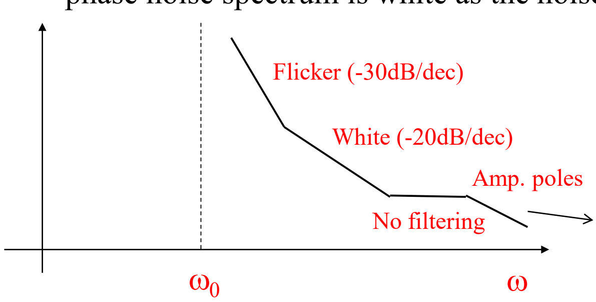
$$S_{\varphi\varphi}(\omega) \propto \left(\frac{\omega_0}{S_F}\right)^2 \frac{1}{\omega^3}$$

- Only white noise is found for frequencies greater than the noise corner frequency (f_{nc}):

$$S_{\varphi\varphi}(\omega) \propto \left(\frac{\omega_0}{S_F}\right)^2 \frac{F \cdot k_B \cdot T}{\omega^2}$$

Phase noise in oscillators - 10

- Noise is no more filtered by the resonant network: the S_F expression used to evaluate phase noise is not valid, and the phase noise spectrum is white as the noise



$$S_{\varphi\varphi}(\omega) = \frac{2 \cdot F \cdot k_B \cdot T}{S}$$

- Finally, above the cut-off frequency of the amplifier block, noise is filtered out by the amplifier transfer function (-20 dB/dec slope if single-pole response)

PLL Phase noise - 1

- Let's suppose to start from lock condition and to inject white noise with B bandwidth
- From PLL transfer functions we get:

$$\varphi_v(s) = H(s) \cdot \varphi_i(s)$$

- and finally the overall phase noise power spectrum is:

$$S^v_{\varphi\varphi}(j\omega) = |H(j\omega)|^2 \cdot S^i_{\varphi\varphi}(s) = |H(j\omega)|^2 \cdot \frac{N}{2SB}$$

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PLL Phase noise - 2

- Output phase noise depends on PLL transfer function (and so from chosen filter):
  - No filter: $$|H(j\omega)|^2 = \frac{(K_p \cdot K_v)^2}{\omega^2 + (K_p \cdot K_v)^2}$$
  - RC filter: $$|H(j\omega)|^2 = \frac{\omega_n^4}{(\omega^2 - \omega_n^2) + 4 \cdot \xi^2 \cdot \omega^2 \cdot \omega_n^2}$$
  - Active filter: $$|H(j\omega)|^2 = \frac{\omega_n^2 \cdot (4 \cdot \xi^2 \cdot \omega^2 + \omega_n^2)}{(\omega^2 - \omega_n^2) + 4 \cdot \xi^2 \cdot \omega^2 \cdot \omega_n^2}$$

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PLL Phase noise - 3

- In particular, if an active filter is considered, for the output phase noise power we get:

$$\overline{\varphi_v^2}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{N}{2SB}\cdot\left|H(j\omega)\right|^2 d\omega = \frac{N}{2SB}\cdot\frac{\omega_n}{2}\cdot\left(\xi+\frac{1}{4\xi}\right)$$

- Therefore, reduction of phase noise is obtained according to the factor ω_n/B ($\xi = 0.5$ is considered):

$$\overline{\varphi_i^2}(t) = \frac{N}{2S}\frac{B_n}{B}$$

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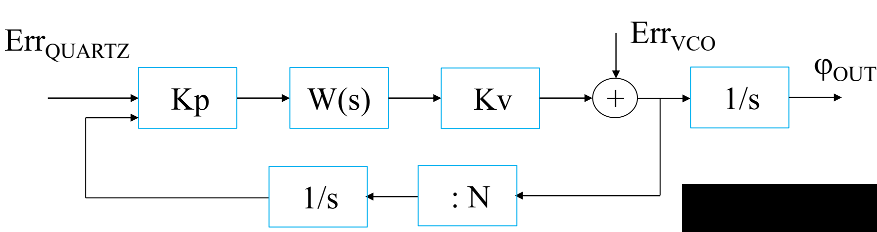
PLL Phase noise - 4



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$$\varphi_{OUT} / Err_{QUARTZ} = \frac{N\cdot(1+sR_2C)}{1+2\xi s/\omega_n+(s/\omega_n)^2}$$

$$\varphi_{OUT} / Err_{VCO} = \frac{N\cdot sR_1C/(Kp\cdot Kv)}{1+2\xi s/\omega_n+(s/\omega_n)^2}$$

- PLL works as LPF for input phase noise, and as a HPF for VCO phase noise: in both cases, ω_n is the cut-off radian frequency

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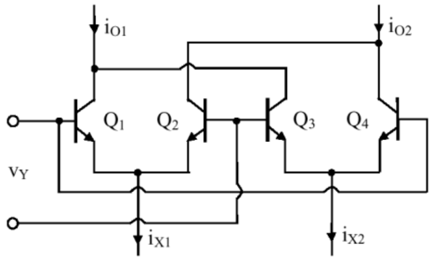
4-quadrant Multiplier

- A 4-quadrant multiplier architecture is obtained by driving the Gilbert cell by means of a differential current i_X



$$i_X = i_{X1} - i_{X2}$$
$$i_O = i_{O1} - i_{O2}$$

$$i_0 = \alpha_F \cdot i_X \cdot \tanh\left(\frac{v_Y}{2V_T}\right)$$

- Also temperature compensation is obtained

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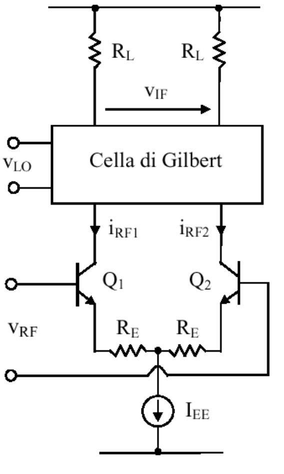
4-quadrant Multiplier

- A multiplier is obtained under small-signal hypothesis:

$$i_X = \alpha_F \cdot I_{EE} \cdot \tanh\left(\frac{v_X}{2V_T}\right) \qquad (R_{EE}=0)$$

$$i_0 = \alpha_F \cdot I_{EE} \cdot \tanh\left(\frac{v_X}{2V_T}\right) \cdot \tanh\left(\frac{v_Y}{2V_T}\right)$$



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4-quadrant Multiplier

- In case of small-signal inputs at the same radian frequency ω:

$$i_0 \cong \alpha_F \cdot I_{EE} \cdot \frac{\hat{v}_X \cdot \cos(\omega t)}{2V_T} \cdot \frac{\hat{v}_Y \cdot \cos(\omega t)}{2V_T}$$

- The tone at radian frequency 2 ω is cancelled by means of a LPF, and the output current is proportional the product of the 2 inputs:

$$i_0 \cong \alpha_F \cdot I_{EE} \cdot \frac{\hat{v}_X \cdot \hat{v}_Y}{(2V_T)^2} \cdot \frac{1 + \cos(2\omega t)}{2}$$

Filtered-out

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4-quadrant Multiplier as Phase Detector

- An analog multiplier and a LPF can be used as a phase detector, for instance in a PLL:
- If the two inputs are at the same frequency but different phase:

$$v_1 = A_1 \cdot \sin(\omega_1 t)$$
$$v_2 = A_2 \cdot \sin(\omega_1 t + \phi)$$

- At the output we get:

$$v_{out} = \frac{A_1 \cdot A_2}{2} \cdot [\cos(\phi) - \cos(2\omega_1 t + \phi)]$$

Filtered-out

- The PD gain Kp depends on amplitude of input signals
- In a PLL, when lock condition is found, $\pi/2$ phase difference is got, as $\cos(\phi) = 0$

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