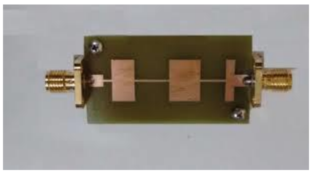
Lumped and Microstrip Filters



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INTRODUCTION

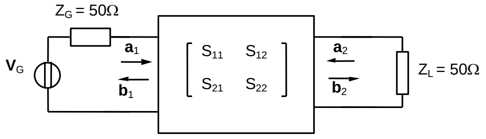
Filters are widely used in microwave systems in order to allow the transmission of signals at desired frequencies and to strongly attenuate signals at undesired frequencies

Therefore, filters, are divided into high pass, low pass, band pass, band stop

An ideal filter should have zero attenuation in the pass band and infinite attenuation in the stopped band Unfortunately, a filter with these characteristics does not exist, so some compromises must be made

Attenuation and Reflection Loss of a two-port network

Consider the following circuit consisting of a two-port network powered by a matched generator (Z_G = 50Ω) and closed on a matched load (Z_L = 50 Ω)



We define the circuit attenuation and return loss as:

$$A_{dB} = 10log_{10}\frac{P_I}{P_0} = 10log_{10}\frac{P_I}{P_E} + 10log_{10}\frac{P_E}{P_0} = A_{RdB} + A_{Ddb}$$

$$L_{RdB} = 10log_{10}\frac{P_I}{P_R}$$

where P_I is the incident power, P_R is the reflected power, P_E is the power entering the network, P_O is the outgoing power. A_{rdB} is the reflection attenuation while A_{DdB} is the dissipation attenuation

With the position V = **a** + **b**, **I** = (**a** - **b**) / Z_0 and having set Z_G = Z_L = Z_0 = 50Ω we have:

$$P_I = \frac{1}{2}\mathrm{Re}\left(\mathbf{V}_I\,\mathbf{I}_I^*\right) = \frac{1}{2Z_0}\left|\mathbf{a}_1\right|^2$$

$$P_R = \frac{1}{2}\mathrm{Re}\left(\mathbf{V}_R\,\mathbf{I}_R^*\right) = \frac{1}{2}\frac{\left|\mathbf{b}_1\right|^2}{Z_0} = \frac{1}{2Z_0}\left|\mathbf{a}_1\right|^2\left|S_{11}\right|^2$$

$$P_E = \frac{1}{2Z_0}\left(\left|\mathbf{a}_1\right|^2 - \left|\mathbf{b}_1\right|^2\right) = \frac{1}{2Z_0}\left|\mathbf{a}_1\right|^2\left(1 - \left|S_{11}\right|^2\right)$$

$$P_O = \frac{-1}{2}\mathrm{Re}\left(\mathbf{V}_O\,\mathbf{I}_O^*\right) = \frac{1}{2Z_0}\left|\mathbf{b}_2\right|^2 = \frac{1}{2Z_0}\left|\mathbf{a}_1\right|^2\left|S_{21}\right|^2$$

It follows that the attenuations and the loss of reflection can be expressed as a function of the two-port network scattering parameters such as:

$$A_{RdB} = 10\log_{10} \frac{1}{1-|S_{11}|^2}$$

$$A_{DdB} = 10\log_{10} \frac{1-|S_{11}|^2}{|S_{21}|^2}$$

$$A_{dB} = 10\log_{10} \frac{1}{|S_{21}|^2}$$

$$L_{RdB} = 10\log_{10} \frac{1}{|S_{11}|^2}$$

Design of a filter by using the low-pass prototype method (LPP)

The low-pass prototype method, also known as the insertion loss method, allows good control of the amplitude and phase characteristics of a filter

If for example it is important to have a low attenuation in the band, a binomial response can be used

if a steep slope (high attenuation out of band) is important, then a Chebyshev response can be used

In all cases, this method allows, within certain limits, to improve the characteristics of the filter by increasing the number of filter elements

Filters are networks (theoretically) without losses; therefore they act as reflection attenuators. Placed $S_{11} = \rho e^{j\varphi}$ we have:

$$A_{dB} = A_{RdB} = 10\log_{10}\frac{1}{1-\rho^2} = 10\log_{10} P_{LR}$$

where:

$$P_{LR} = \frac{1}{1-\rho^2}$$

P_{LR} is the Power Loss ratio.
If we consider a real signal v(t) and we denote its Fourier transform by V(f) , it results V(f) = V *(-f) (even real part and odd imaginary part, or even module and odd phase), the same holds for i(t) and therefore
I (f) = I*(- f). From these properties it results
Z (f) = V(f) /I (f) = Z *(- f) and Γ(f) = Γ *(- f), so, in conclusion, we have that ρ is an even function of ω and even more ρ^2

Placed:

$$\rho^2 = \frac{M(\omega^2)}{M(\omega^2)+N(\omega^2)}$$

We have:

$$P_{LR} = \frac{1}{1-\rho^2} = \frac{1}{1-\frac{M}{M+N}} = \frac{M+N}{M+N-M} = \frac{M+N}{N}$$

and in conclusion:

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

Different frequency trends can be assigned to the M and N polynomials. These trends identify the filter family. In the LPP method initially we consider M and N as a function of a normalized pulsation ω' (dimensionless)

Binomial or Maximally Flat or Butterworth Filters

For these filters we choose:

$$N(\omega'^2) = 1; \quad M(\omega'^2) = K^2\,\omega'^{2N}$$

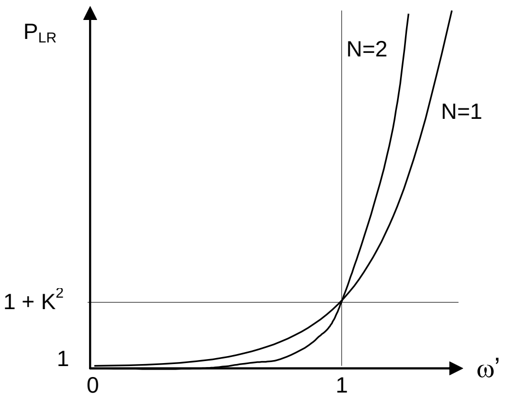
where K^2 is a constant called filter tolerance and N is called filter order.

So we have:

$$P_{LR} = 1 + K^2\omega'^{2N}$$

$$A_{dB} = 10\log_{10}[1 + K^2\omega'^{2N}]$$

The figure shows the qualitative trend of the PLR for N = 1 and N = 2



For $\omega' = 0$ the $P_{LR}(\omega')$ function has the first $(2N-1)$ derivatives equal to zero

for $\omega' = 1$ (cut-off) it results:

$$P_{LR} = 1 + K^2$$

$$A_{CdB} = 10 \log 10 (1 + K^2)$$

$$K^2 = 10^{Ac/10} - 1$$

This last equation shows that the filter tolerance (K^2) is linked to the cut-off attenuation. For example for $K^2 = 1$ we have $A_{CdB} = 3$dB.

for $\omega' \gg 1$ it results:

$$P_{LR} = K^2 \omega'^{2N}$$

$$A_{dB} = 10\log 10\ K^2 + 10\log 10\ (\omega'^{2N}) = 10\log 10\ K^2 + 20N\log 10\ \omega'$$

The attenuation increases by 20 N dB per decade. Therefore, N determines the slope of the filter and it is chosen to have a certain attenuation out of band

Constant Ripple or Chebyshev filters

For these filters we choose:

$$N(\omega'^2) = 1; \quad M(\omega'^2) = K^2\ T_N^2(\omega')$$

where $T_N(\omega')$ are the Chebyshev polynomials of degree N:

$$T_N(\omega') = \cos[N\cos^{-1}(\omega')] \qquad \text{for } \omega' < 1$$

$$T_N(\omega') = \cosh[N\cosh^{-1}(\omega')] \qquad \text{for } \omega' \geq 1$$

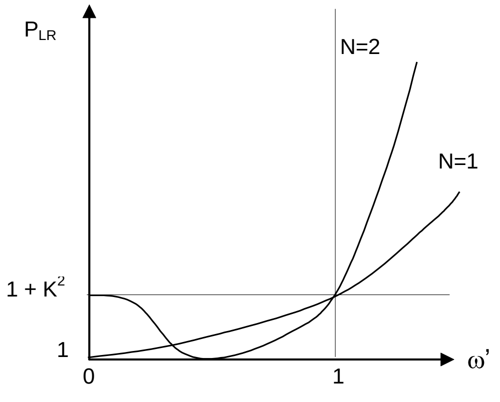
Chebyshev polynomials have the following properties: $T_N(0) = 0$ for odd N, $T_N(0) = 1$ for even N, T_N oscillates between ± 1 for $\omega' < 1$ while it grows monotonically for $\omega' > 1$.
For $\omega' \gg 1$ holds the approximation: $T_N^2(\omega') = (1/4)(2\omega')^{2N}$

For these filters we have:

$$P_{LR} = 1 + K^2\ T_N^2(\omega')$$

$$A_{dB} = 10\log_{10}[1 + K^2\ T_N^2(\omega')]$$

The qualitative trend of the P_{LR} for N = 1 and N = 2 is shown in the following figure



For ω' = 1 (cut-off) it results:

$$P_{LR} = 1 + K^2$$

$$AC_{dB} = 10 \log_{10} (1 + K^2)$$

for $\omega' \gg 1$ it results:

$$P_{LR} = (1/4)\, K^2\, (2\omega')^{2N}$$

$$A_{dB} \approx 10\log10\, K^2 + 10\log_{10} (\omega'^{2N}) + 10\log_{10} [(1/4)\, 2^{2N}] =$$

$$10\log_{10} K^2 + 20N\log10\omega' + 10\log_{10} [(1/4)\, 2^{2N}]$$

The attenuation increases by 20 N dB per decade, but is (1/4) (2^{2N}) times greater than that of the binomial filter

Transformation formulas

The functions that have been considered for the response of the PBPR filter are normalized in terms of frequency

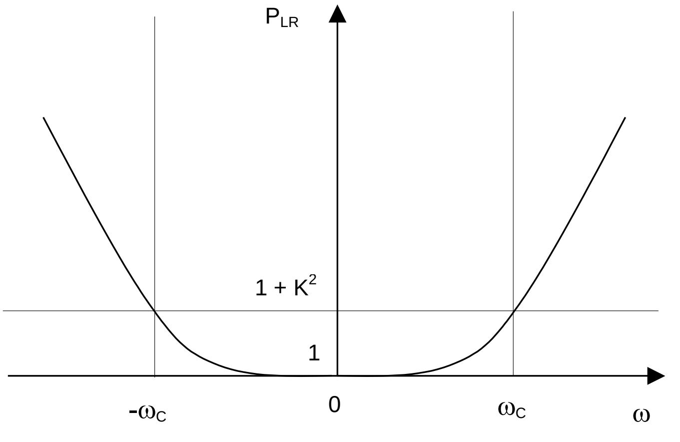
Starting from the LPP we may get a low pass filter with cut-off pulsation given by ω_c with the following transformation:

$$\omega' = \frac{\omega}{\omega_C}$$

For example, with reference to a binomial filter, denormalizing with respect to frequency, we obtain:

$$P_{LR} = 1 + K^2 \left(\frac{\omega}{\omega_C}\right)^{2N}$$

The qualitative response trend for a fixed N is shown in the following figure. The Figure shows that for $\omega = \omega_c$ we have PLR = 1 + K^2 and, for $\omega = 0$ we have PLR = 1.



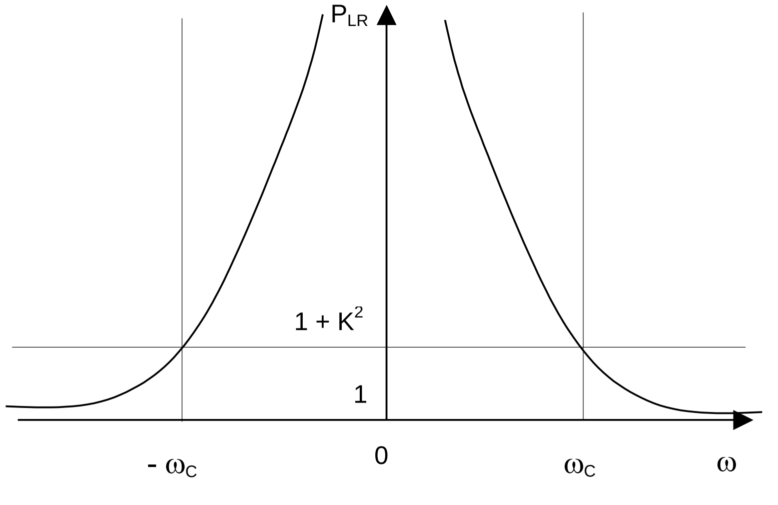
Starting from the LPP we may get a high pass filter with cut-off pulsation given by ω_c with the following transformation:

$$\omega' = -\frac{\omega_C}{\omega}$$

For example with reference to a binomial filter we get:

$$P_{LR} = 1 + K^2 \left(-\frac{\omega_C}{\omega}\right)^{2N}$$

A qualitative response of the filter, as a function of frequency and for a fixed N is shown in the following figure. In this case, for $\omega = \infty$ we have $P_{LR} = 1$; for $\omega = \omega_c$ we have $P_{LR} = 1 + K^2$ and finally for $\omega = 0$ we have $P_{LR} = \infty$



Starting from the LPP we may get a band pass filter with lower cut pulsation given by ω_1 and higher cut pulsation given by ω_2 with the following transformation:

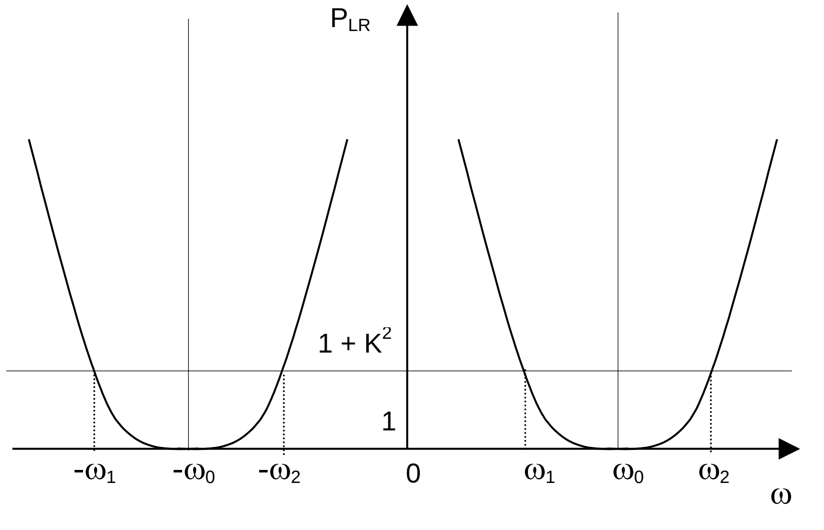
$$\omega' = \frac{\omega_0}{(\omega_2 - \omega_1)} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

with $\omega_0 = \sqrt{\omega_1 \omega_2}$

For example with reference to a binomial filter we get:

$$P_{LR} = 1 + K^2 \left[\frac{\omega_0}{(\omega_2 - \omega_1)} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]^{2N}$$

The qualitative frequency trend of the filter for a given N is shown in the following figure



for $\omega = \omega_0$ we have $P_{LR} = 1$

for $\omega = \omega_2$ we have

$$P_{LR} = 1 + K^2 \left[\frac{\omega_0}{(\omega_2 - \omega_1)} \left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right)\right]^{2N} = 1 + K^2$$

for $\omega = \omega_1$ we have $P_{LR} = 1 + K^2$

for $\omega = 0$ we have $P_{LR} = \infty$

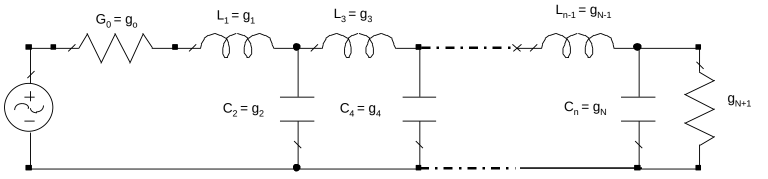
Circuit realization of the filter

The frequency response of the LPP filter can be obtained with electric circuits made by lumped elements inductors and capacitors

Below are two possible circuits consisting of a cascade of inductors and capacitors in a number equal to the order of the filter

In particular it can be noticed that, since the circuit must have a low-pass type behavior, there are always series inductors and parallel capacitors

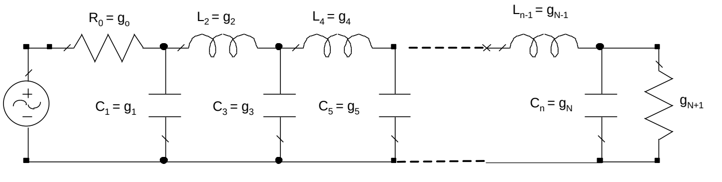
The network in the following figure, whose components are dimensionless normalized quantities, begins with a series inductor



By progressively numbering the elements with an index k from left to right, the network has inductors for odd k and capacitors for even k, and ends in two different ways (capacitors or inductors) depending on the total number N of elements of the filter

It can also be noted that impedances and admittances alternate in the network; therefore if g_N is a capacitor in parallel (admittance) $g_N + 1$ is a resistance; if g_N is an inductor in series (impedance) $g_N + 1$ is a conductance

Another possible circuit with a LPP filter response is that in the figure below. In this case the first element of the network is a capacitor



Also in this case all the components of the network are dimensionless normalized quantities

The network has capacitors for odd k and inductors for even k, and ends in two different ways (capacitor or inductor) depending on the value of N

Dimensioning of the prototype filter

The design specifications of a filter require, once the type of response is chosen, the assignment of a certain attenuation out of band

For example, for $\omega' = 2$, the attenuation must be $A_{dB} = 20dB$
(for a real PB that means $A_{dB} = 20dB$ at $\omega = 2\ \omega_c$)

Furthermore, the cut-off attenuation of the binomial filter or the ripple for the Chebyshev filter must be assigned. For example for $\omega' = 1$ the attenuation must be $A_{dB} = 3dB$
(for a real PB that means to have $A_{CdB} = 3dB$ at $\omega = \omega_c$)

For the sizing of the filter it is possible to proceed analytically observing that having A_{CdB} ($\omega' = 1$) = 3dB means K = 1; at this point, with reference to a binomial filter, we have $A_{dB} = 10\log_{10}[1 + (\omega')^{2N}]$ and we can vary N until we have $A_{dB}(\omega' = 2) > 20dB$. In this way, proceeding by step, we can find N

Finally we can use analytical formulas present in literature (*) which, given N, allow us to calculate $g_0, g_1, \ldots, g_{N+1}$

This procedure is too rigorous and it is easier to use graphs or tables

In particular, the graph in Fig. 1 allows to determine the number of elements of a binomial filter based on the specifications

(*) G.L. Mattei, L. Young and E.M.T. Jones, Microwave filters, Impedance-Matching Networks and Coupling Structrures, Artech House, 1980

Binomial Filter, A_{CdB} = 3

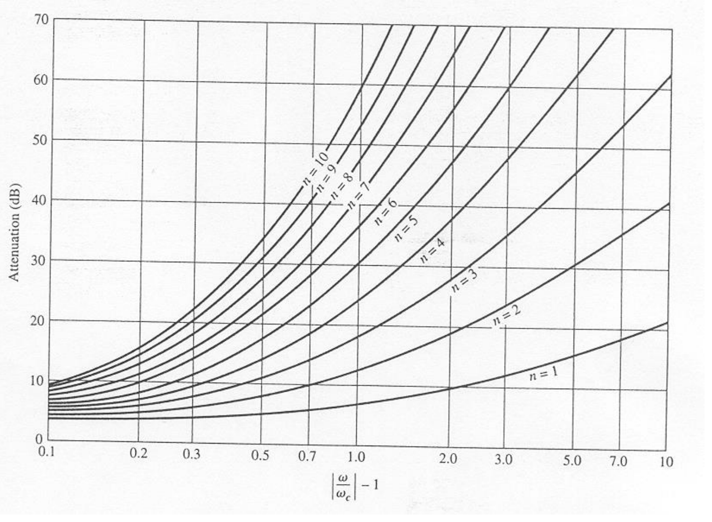


Figure 1

The graph in Figure refers to the case A_{CdB} = 3dB; on specific texts, such as the one mentioned above, there are similar ones for other A_{CdB} values

In the considered example we have ω' = 2 and therefore ω' - 1 = 1. From the graph of Fig. 1 we see that to have A_{dB} > 20 dB, we have to choose N = 4

Using the table in Fig. 2 (valid for A_{CdB} = 3dB) the values of the 4 reactive elements (g_1, g_2, g_3, g_4) that make up the filter can be obtained

For binomial filters it is always g_0 = 1 and g_N + 1 = 1

Binomial Filter, $A_{CdB} = 3$

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Figure 2

If the PLR of the filter thus obtained is evaluated, we found:

$$P_{LR}(\omega') = 1 + (\omega')^8$$

that is analogous to that theoretically fixed for binomial filters

A similar approach can be used for the design of Chebyshev filters

The corresponding graphs and tables are shown in Fig. 3 and Fig. 4 for filters with ripple of 0.5 and 3 dB respectively

For these filters we have $g_0 = 1$ while $g_N + 1$ is equal to 1 for odd N but different from 1 for even N

Chebyshev Filter, A_{CdB} = 0.5

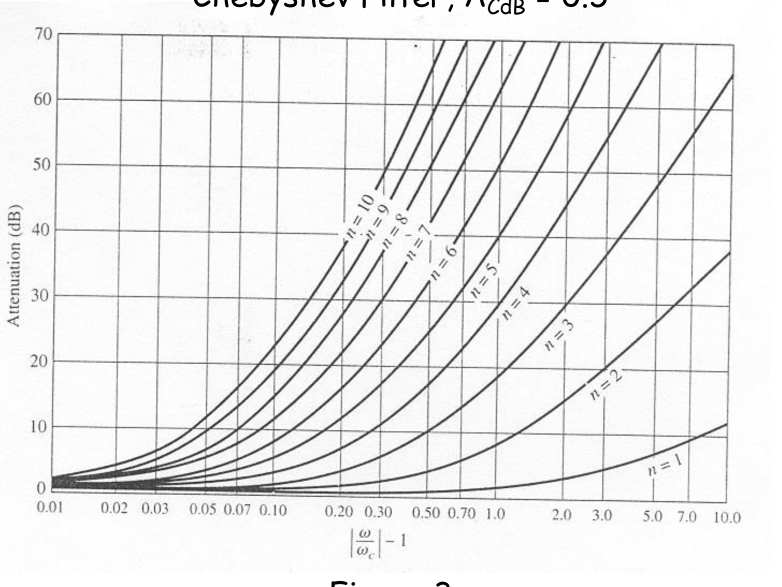


Figure 3

Chebyshev Filter, A_{CdB} = 3

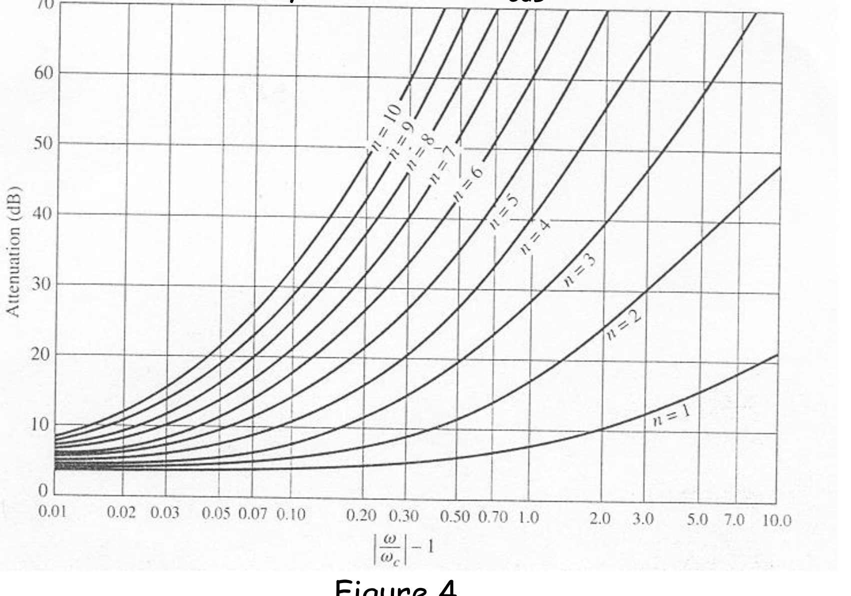


Figure 4

Chebyshev Filter, A_{CdB} = 0.5

(a) N	g_1	g_2	g_3	g_4	0.5 dB g_5	Ripple g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	1.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.5696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7329	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5329	0.8842	1.9841

Figure 5

Chebyshev Filter, A_{CdB} = 3

N	g_1	g_2	g_3	g_4	3.0 dB g_5	Ripple g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Figure 6

Dimensioning of the real filter

In the LPP circuit the reactances are dimensionless (normalized with respect to R_0). To obtain physical quantities they must be denormalized with respect to the impedance, as well as, with respect to pulsation

This operation can be performed in two consecutive steps. In the first step we denormalized with respect to the impedance and in the second with respect to the frequency

The denormalization with respect to the impedance is carried out by multiplying or dividing the dimensionless parameter by the reference impedance $R_0 = 50\Omega$:

$$L'_K = R_0 L_K = R_0 g_K \ [\Omega]$$

$$C'_K = \frac{C_K}{R_0} = \frac{g_K}{R_0} \ [S]$$

$$R'_K = R_K R_0 = g_K R_0 \ [\Omega]$$

$$G'_K = \frac{G_K}{R_0} = \frac{g_K}{R_0} \ [S]$$

Denormalization with respect to frequency is carried out using the transformations previously introduced

Low pass filters

For the low pass circuit we have $\omega' = \frac{\omega}{\omega_C}$ so:

$$X'_K = \omega' \, L'_K = \frac{\omega}{\omega_C} L'_K \;\; [\Omega]$$

this is equivalent to having an inductance:

$$L''_K = \frac{L'_K}{\omega_C} = \frac{R_0 \, L_K}{2\,\pi f_C} = \frac{R_0 \, g_K}{2\,\pi f_C} \;\; [H]$$

Similarly we have:

$$B'_K = \omega' \, C'_K = \frac{\omega}{\omega_C} C'_K \;\; [\Omega]$$

and this is equivalent to a capacitance

$$C''_K = \frac{C'_K}{\omega_C} = \frac{C_K}{R_0 2\,\pi f_C} = \frac{g_K}{R_0 2\,\pi f_C} \;\; [F]$$

In this way the capacitance and inductance values to be inserted in the filter network are univocally determined

High pass filters

With reference to the high pass we have: $\omega' = -\frac{\omega_C}{\omega}$
from which follows:

$$X'_K = \omega' \, L'_K = -\frac{\omega_C}{\omega} L'_K \; [\Omega]$$

This reactance is equivalent to a capacitance of value:

$$C''_K = \frac{1}{L'_K \omega_C} = \frac{1}{R_0 \, g_K 2\pi f_C} \;\; [F]$$

So the inductor turns into a capacitor.
Likewise, for the capacitor we have:

$$B'_K = \omega' \, C'_K = -\frac{\omega_C}{\omega} C'_K \; [\Omega]$$

This susceptibility corresponds to that of an inductor, therefore the capacitor is transformed into an inductor of value:

$$L''_K = \frac{1}{L'_K \omega_C} = \frac{R_0}{g_K 2\pi f_C} \; [H]$$

Band pass filters

With reference to the band pass, we have: $\omega' = \frac{\omega_0}{(\omega_2 - \omega_1)}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$
and therefore we have:

$$X'_K = \omega' \, L'_K = \frac{\omega_0}{(\omega_2 - \omega_1)}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) L'_K =$$

$$= \frac{\omega \, L'_K}{(\omega_2 - \omega_1)} - \frac{\omega_0^2}{(\omega_2 - \omega_1)} \frac{L'_K}{\omega} = \omega \, L''_K - \frac{1}{\omega C''_K}$$

where:

$$L''_K = \frac{L'_K}{\omega_2 - \omega_1} = \frac{R_0 \, g_K}{\omega_2 - \omega_1} \ [H]$$

$$C''_K = \frac{\omega_2 - \omega_1}{\omega_0^2 \, L'_K} = \frac{\omega_2 - \omega_1}{\omega_0^2 \, R_0 \, g_K} \ [F]$$

therefore the inductor is transformed into a series of a capacitor and an inductor

Similarly, starting from:

$$B'_K = \omega' \, C'_K = \frac{\omega_0}{(\omega_2 - \omega_1)}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) C'_K$$

We found:

$$C''_K = \frac{g_K}{R_0(\omega_2 - \omega_1)} \ [F]$$

$$L''_K = \frac{(\omega_2 - \omega_1) R_0}{\omega_0^2 \, g_K} \ [H]$$

Thus the capacor is transformed into the parallel of a capacor and an inductor

Suppose we want to design a **maximally flat low pass filter** with a cutoff frequency of 2 GHz and an attenuation of at least 15 dB at a frequency of 3 GHz. We want to calculate and graph the amplitude response for frequencies from 0 to 4 GHz, and compare it with a filter of the same order with constant ripple (3.0 dB ripple)

Initially the order N satisfying the specification of the attenuation at 3 GHz is evaluated. Since we have at 3 GHz $\frac{\omega}{\omega_C} - 1 = 0.5$, from Fig. 1 we see that N = 5 is sufficient. Then from the table in Fig. 2 the values of the elements of the prototype are obtained:

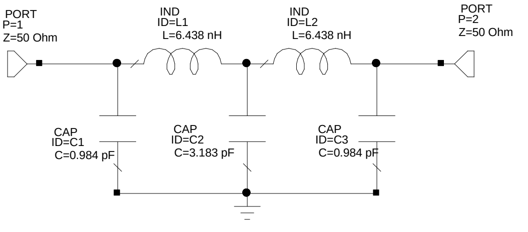
$g_1 = 0.618$, $g_2 = 1.618$, $g_3 = 2.000$, $g_4 = 1.618$, $g_5 = 0.618$

Thus the previous equations can be used to obtain the denormalized values of the elements for the circuit and we obtain:

$$C''_1 = 0.984 \text{ pF}, L''_2 = 6.438 \text{ nH}, C''_3 = 3.193 \text{ pF},$$

$$L''_4 = 6.438 \text{ nH}, C''_5 = 0.984 \text{ pF}$$

The final filter circuit was implemented on a commercial CAD (MicrowaveOfficeTM) and is shown in the following figure



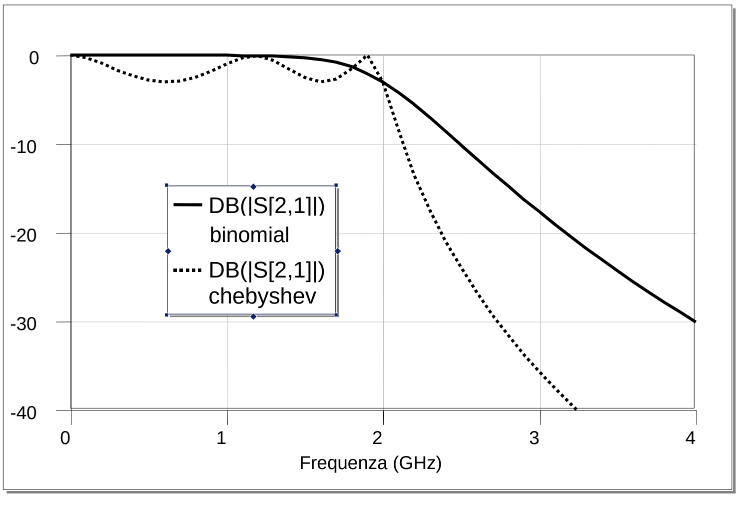
Similarly, the values of the components for the constant ripple filter, for N = 5, can be determined from Fig. 3 and we have:

$$C''_1 = 5.539 \text{ pF}, L''_2 = 3.024 \text{ nH}, C''_3 = 7.220 \text{ pF},$$

$$L''_4 = 3.024 \text{ nH}, C''_5 = 5.539 \text{ pF}$$

The results of the amplitude for these two filters, obtained with the CAD, are shown in the next figure. These results clearly show the compromises associated with the two types of filters.

The amplitude response of the constant ripple filter has the best slope at the cut-off. The maximally flat filter response has the flattest attenuation characteristics in the pass band, but a slightly lower slope at the cut off



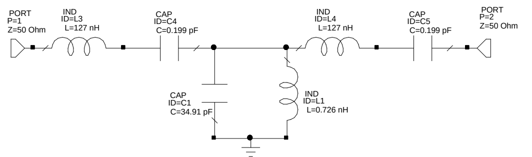
Suppose we want to design a **band pass filter that has a constant ripple response of 0.5 dB**. The central frequency is 1 GHz, the bandwidth is 10% (f_1 = 950 MHz, f_2 = 1050 MHz, f_0 = 1 GHz), moreover, A = 15 dB at f = 1.1 GHz.
We have ω' = 2 and N = 3. From the table in Fig. 4 we have that the values of the elements of the prototype circuit are:

$$g_1 = 1.5963,\ g_2 = 1.0967,\ g_3 = 1.5963$$

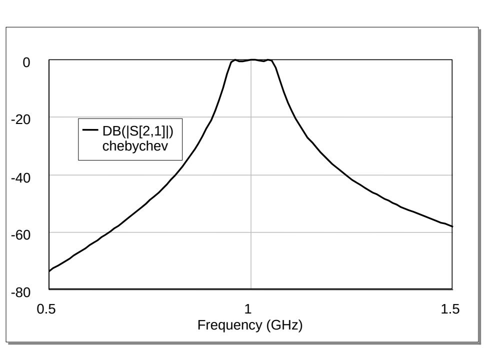
Thus the design equations give the following capacitance and inductance values

L''_1 = 127.0 nH $\qquad$ C''_1 = 0.199 pF
L''_2 = 0.726 nH $\qquad$ C''_2 = 34.91 pF
L''_3 = 127.0 nH $\qquad$ C''_3 = 0.199 pF

The circuit was implemented with MicrowaveOfficeTM and is shown in the Fig (a). The amplitude response obtained with the CAD is shown in Fig. (b)



(a)



(b)

Filter Realization on microstrip

Filters seen so far work well at low frequencies where it is possible to realize the desired values of L and C with lumped elements. At high frequencies inductances and capacities are realized with distributed elements

With reference to the microstrip technology, low-pass filters can be realised based on Richard's transformations and Kuroda's identities or using short low and high impedance lines that behave like series inductances or parallel capacitances

With other techniques it is also possible to realise high pass filters, and band pass filters. However, we are not going to deal with these techniques and they are properly addressed in other textbooks, see for example:

G.L. Mattei, L. Young and E.M.T. Jones, Microwave filters, Impedance-Matching Networks and Coupling Structrures, Artech House, 1980

Commensurate filters

Richard Transformations

Richard's transformations are based on the relations of the normalized impedances of a line section closed on a short circuit or on an open circuit, given by:

$$\hat{Z}_{IN} = j\hat{Z}_C \tan(\beta l) = j\hat{Z}_C \tan(\theta)$$

$$\hat{Y}_{IN} = j\hat{Y}_C \tan(\beta l) = j\hat{Y}_C \tan(\theta)$$

In the above equations we can choose 'l' in order to obtain the desired values of $\hat{Z}_{IN}$ and $\hat{Y}_{IN}$ and therefore of L_K and C_K; it is more appropriate, however, to work with commensurate lines, that is, with lines having all the same length (this makes the filter response periodic)

Then we set l = $\frac{\lambda}{8}$ (for $\omega = \omega_C$) from which it follows:

$$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{8} = \frac{\pi}{4}$$

and therefore:

$$\tan(\beta l) = 1$$

So choosing

$$\hat{Z}_{IN} = j\,\hat{Z}_C = j\,\omega'\,L_K = j\,\omega'\,g_K$$

$$\hat{Y}_{IN} = j\,\hat{Y}_C = j\,\omega'\,C_K = j\,\omega'\,g_K$$

At the cut-off we have:

$$\hat{Z}_{IN} = j\,\hat{Z}_C = j\,L_K = j\,g_K$$

$$\hat{Y}_{IN} = j\,\hat{Y}_C = j\,C_K = j\,g_K$$

And in conclusion we can set:

$$\hat{Z}_C = L_K = g_K$$

$$\hat{Y}_C = C_K = g_K$$

$\hat{Z}_{IN}$ coincides with the normalized impedance of an inductor of value $L_K = g_K$ at the normalized pulsation $\omega' = 1$ ($\hat{Z}_{IN} = j\omega' L_K$)

$\hat{Y}_{IN}$ coincides with the normalized admittance of a capacior with value $C_K = g_K$ at the normalized pulsation $\omega' = 1$ ($\hat{Y}_{IN} = j\omega' C_K$)

Then, denormalizing with respect to the impedance R_0 we have:

$$Z_C = g_K R_0$$

$$Y_C = g_K / R_0$$

The above relations underline the possibility of realizing inductance and capacitance values by simply using $\frac{\lambda}{8}$ lines closed on open or on short circuit with a suitable characteristic impedance which is linked to the g_K coefficients of the LPP filter

Obviously the circuit is equivalent to the theoretical one only for $\omega' = 1$ ($\omega = \omega_C$); moving away from ω_C the response differs from that of the theoretical filter

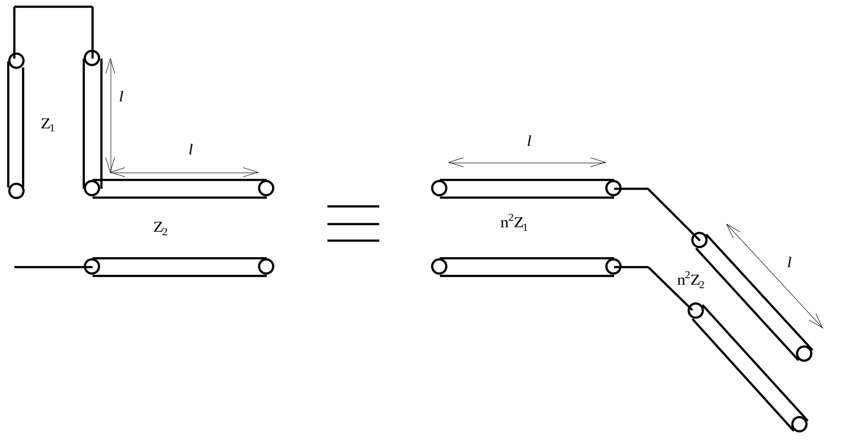
Kuroda Identity

Kuroda's identity allows to transform series stubs into parallel stubs. In fact, stubs in parallel are simpler to make in microstrip

In particular it can be easily demonstrated by comparing the ABCD matrices of the two circuits of the following figure, that a series stub of length 'l' with characteristic impedance Z_1 closed on a short circuit, followed by a section of line with characteristic impedance Z_2 of length 'l', is equivalent to a section of line with characteristic impedance $n^2 Z_1$ with

$$n^2 = 1 + Z_2 / Z_1$$

of length 'l', followed by an open stub in parallel with characteristic impedance $n^2 Z_2$ of length 'l' and terminated on an open circuit

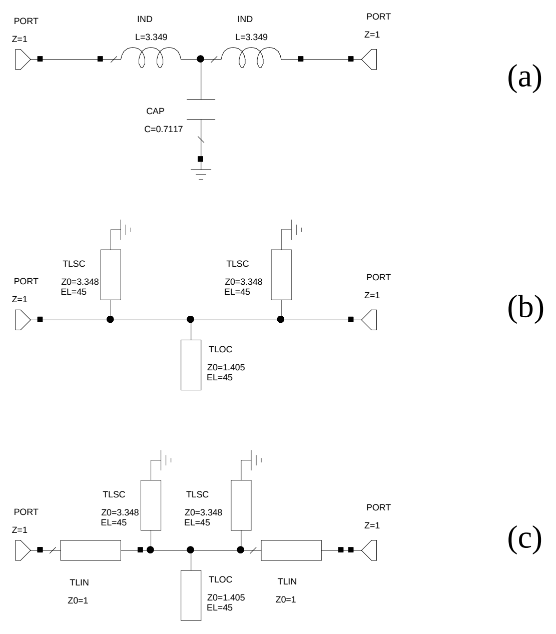


Suppose we want to design a low pass filter using microstrip lines. The specifications are: 4 GHz cut-off frequency, third order, and a 3 dB constant ripple characteristic

From the table of Fig. 6 the normalized values of the elements of the low pass prototype are:

$$g_1 = 3.3487,\ g_2 = 0.7117,\ g_3 = 3.3487$$

which are assigned to the circuit shown in figure (a) below. The next step is to use Richard's transformation to convert the inductors in series into stubs in series and the capacitors in parallel into stubs in parallel, as shown in Fig. (b)



According to the previous described theory, the normalized characteristic impedance of the stub in series (inductor) is g_1, and the normalized characteristic impedance of a stub in parallel (capacitor) is $1/g_2$

For the synthesis of commensurate lines, all the stubs are $\frac{\lambda}{8}$ long at $\omega = \omega_C$ (it is generally very convenient to work with normalized quantities up to the last step in the project)

The stubs in series of Fig. (b) would be very difficult to implement in microstrip, so Kuroda's identity is used to transform them into parallel stubs

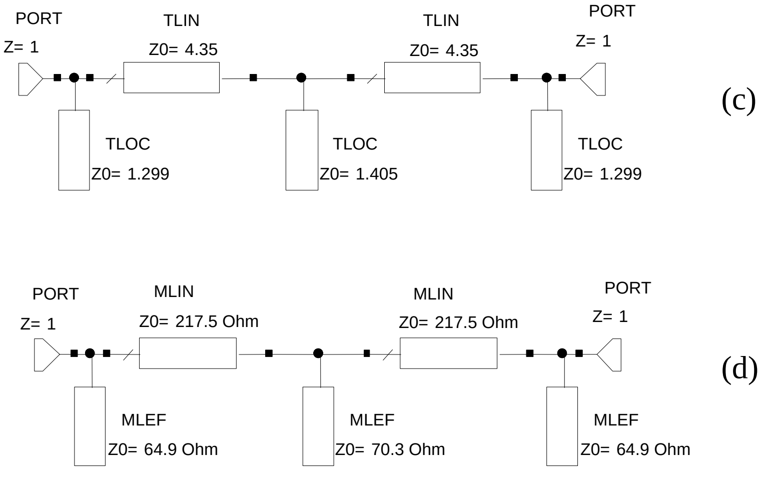
First, unitary elements are added to both ends of the filter, as shown in Fig. (c)

These redundant elements have no effect on the performance of the filter because they are adapted with the source and the load (Z_0 = 1)

We can then apply Kuroda's identity as seen above to both ends of the filter. In both cases we have:

$$n^2 = 1 + \frac{Z_2}{Z_1} = 1 + \frac{1}{3.3487} = 1.299$$

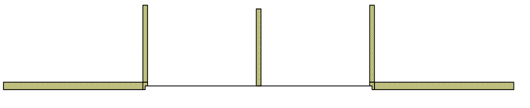
The result is shown in the Fig (c)



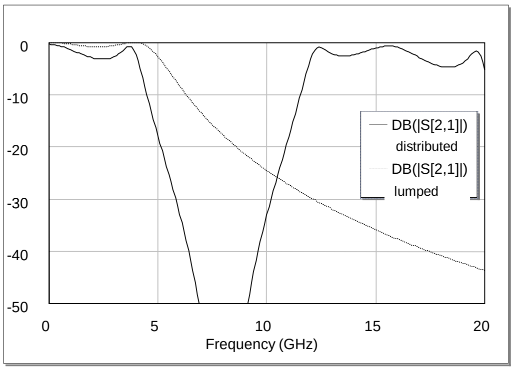
Finally, the circuit is scaled in frequency and impedance, which simply implies multiplying the normalised impedances by 50Ω, and fixing the lines and stubs long λ/8 at 4 GHz. The final circuit is shown in Fig. (d), and the corresponding microstrip layout in Fig. (e) (for a 254 μm RO4003 substrate)

The calculated amplitude response is plotted in Fig. (f), together with the response achieved by using lumped elements. It can be noted that the features are very similar under 4 GHz, but the distributed element filter has a more defined cut off

Extending the frequency analysis, we note that the distributed element filter has a response that is repeated every 16 GHz, as a consequence of the periodic nature of Richard's transformation



(e)



(f)

Step filters

An alternative technique for the synthesis of a low pass filter is to use a scale network. This network is based on the following property

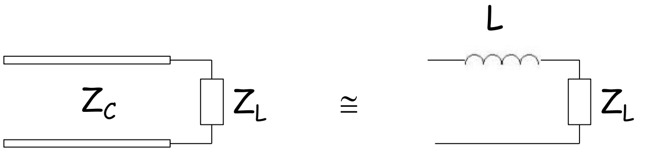
The input inpedance of a line of lenght l is given by:

$$Z_{IN} = Z_C \frac{Z_L \cos\beta l + jZ_C\, sen\beta l}{Z_C \cos\beta l + jZ_L\, sen\beta l}$$

For a short line (l << λ / 10) closed on a load with a magnitude impedance much lower than the line characteristic impedance($|Z_L|$ << Z_C) we have:

$$Z_{IN} = Z_C \frac{Z_L + jZ_C\beta l}{Z_C + jZ_L\beta l} = Z_L + jZ_C\beta l = Z_L + j\omega l$$

The line behaves like an inductor in series with the load and with value L = Z_Cl / c



Similarly, a short line closed on a load with a magnitude impedance is much greather than the line characteristic impedance($|Z_L| \gg Z_C$) behaves like a capacitor in parallel with the load and with value $C = Y_C l / c$

For example, one can suppose for low-impedance lines $Z_C = Z_L$ and for high-impedance lines $Z_C = Z_H$

Therefore, with reference to capacitor, we have:

$$C = \frac{Y_C l}{c} = \frac{1}{Z_L}\frac{1}{c}$$

Fixed Z_L, the only parameter to act on is l, and we have:

$$\theta = \beta l = \beta\, C\, Z_L\, c = \frac{\omega_c}{c}\, C\, Z_L\, c = \omega_c C\, Z_L\, c$$

But: $C = \frac{g_K}{R_0 \omega_c}$

then:

$$\beta l = \frac{\omega_c\, g_K Z_L}{R_0 \omega_c} = \frac{g_K Z_L}{R_0}$$

Similarly for the inductors we have:

$$\beta l = \frac{g_K R_0}{Z_H}$$

Suppose we want to design a **low pass filter that has a maximally flat response** and a 5.5 GHz cutoff frequency and that has more than 10dB of attenuation at 7 GHz

Suppose further that the highest line impedance practically available (Z_H) is 75 Ω, and the lowest (Z_L) 15 Ω
At 7 GHz we have:

$$\frac{\omega}{\omega_C} - 1 = \frac{7}{5.5} - 1 = 0.273$$

therefore Fig. 2 tells us that N = 5 provides the necessary attenuation at 7 GHz

From the Table of Fig. 2 the values of the elements of the prototype are obtained:

$$g_1 = 0.618,\ g_2 = 1.618,\ g_3 = 2,\ g_4 = 1.618,\ g_5 = 0.618$$

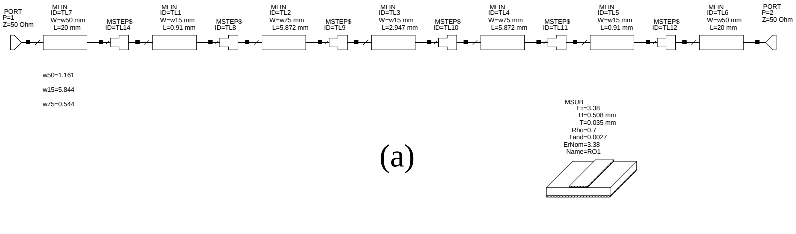
Subsequently, using the previous results we have:

$$\beta l_1 = \frac{g_1 R_0}{Z_H} = 10.62° \qquad \beta l_2 = \frac{g_2 Z_L}{R_0} = 61.80° \qquad \beta l_3 = \frac{g_3 R_0}{Z_H} = 34.38°$$

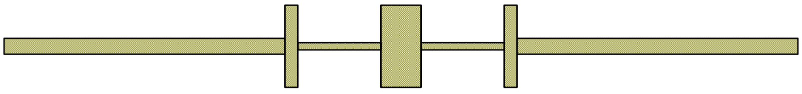
$$\beta l_4 = \frac{g_4 Z_L}{R_0} = 61.80° \qquad \beta l_5 = \frac{g_5 R_0}{Z_H} = 10.62°$$

The physical circuit is shown in the following Fig (a) while the final realization on microstrip is shown in Fig. (b) where the wide sections have Z_L = 15 Ω while the narrow ones Z_H = 75 Ω

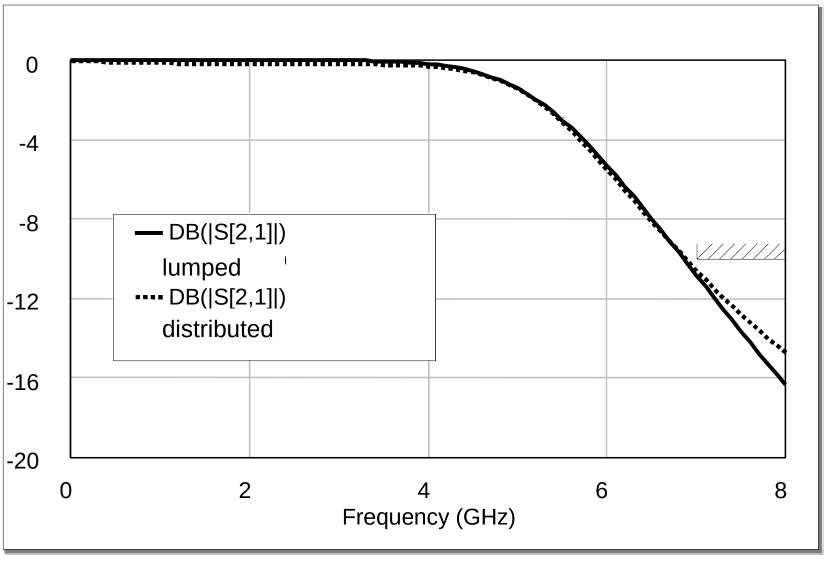
Note that Fig. (c) shows the filter attenuation compared with the attenuation of a corresponding filter with discrete elements. The trends in the pass band are very similar, but the circuit with discrete elements has greater attenuation at higher frequencies. This is due to the fact that the elements of the step filter deviate significantly from the values of the discrete elements at higher frequencies. The step filter can also have other bands passing at higher frequencies, but the response will not be perfectly periodic because the lines are not commensurate



(a)



(b)


0
-4
-8
-12
-16
-20
0
2
4
6
8
Frequency (GHz)
DB(|S[2,1]|)
lumped
DB(|S[2,1]|)
distributed

(c)