Chapter 4
Transmission Line Transformers and Hybrids

Introduction

\[ Z_0 \]
\[ Z_s \]
\[ Z_L \]
\[ l \]

Figure 1. Transmission line parameters.

For a transmission line of characteristic impedance \( Z_0 \) and length \( l \), the sending end impedance is given by:

\[
Z_S = \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}
\]

Eqn. 1

If the transmission line is close to one quarter wavelength long then \( \tan(\beta l) \gg 1 \) so that:

\[
Z_S = \frac{jZ_0 \tan(\beta l)}{jZ_L \tan(\beta l)} = \frac{Z_0^2}{Z_L}
\]

Eqn. 2

so that: \( Z_SZ_L = Z_0^2 \) or \( Z_S = \frac{Z_0^2}{Z_L} \)

Eqn. 3

The impedance transformation ratio \( R \) is given by:

\[
R = \frac{Z_0}{Z_S} \quad \text{or} \quad Z_S = \frac{Z_0}{R} \quad \text{and} \quad Z_L = RZ_0
\]

Eqn. 4

If the allowable Voltage Standing Wave Ratio (VSWR) is \( W \), so that the limiting value of the sending end impedance is \( WZ_0 \), substituting equation 4 into equation 1 gives:

\[
WZ_S = W \frac{Z_0}{R} = Z_0 \left[ \frac{RZ_0 + jZ_0 \tan(\beta l)}{Z_0 + jRZ_0 \tan(\beta l)} \right]
\]

Eqn. 5

Solving for \( \tan(\beta l) \) and using the fact that normally \( W^2 \ll R^4 \) gives:

\[
\tan(\beta l) = \frac{R^4 - W^2}{\sqrt{R^2(2W^2 - 1)}} \approx R \sqrt{\frac{1}{W^2 - 1}}
\]

Eqn. 6

The higher \( R \), the higher \( \tan(\beta l) \) must be and thus the narrower the bandwidth for a given VSWR. This can easily be illustrated with an example.

Example:

If the allowable VSWR is 1.2, that is for a 50 Ω system the impedance should be between 60 Ω and 41.6 Ω, and if \( R = 2 \) (i.e. a 4:1 impedance transformation) then \( \beta l = 70.8^\circ \) i.e. \( l = 0.197 \lambda \). The bandwidth is thus \( 2(90-70.8)/90 = 43\% \)
If R is changed to $R = \sqrt{2}$ (i.e. a 2:1 impedance transformation) then the same total impedance transformation can be achieved by having two transmission line sections in cascade. If the load impedance is $Z_L$ then the impedance at the end of the first section is $2Z_L$ and the impedance at the end of the second section is $4Z_L$. The total length of the transformer is thus half a wavelength.

Solving for $\beta l$ gives $\beta l = 59.6^\circ$ i.e. $l = 0.166\lambda$. The bandwidth is thus $2(90-59.6)/90 = 67.5\%$

For very high frequencies, many sections can be cascaded and the line will have an exponential change of impedance for each of the quarter wavelength sections. This can readily be produced using RF PCB technology.

Figure 2 shows a plot of the input impedance variation of a transmission line transformer, transforming a 12.5 $\Omega$ load into a 50 $\Omega$ load. For the one line transformation, the impedance transformation is done in one transmission line section with a characteristic impedance of 25 $\Omega$. The Magenta curve is for a 2-line transformation, with the impedance transformation ratio $R = \sqrt{2}$ for each line.

If the impedance transformation ratio of the two lines in a two line impedance transformer is changed from the $R = \sqrt{2}$ for each of the lines, such that the total impedance transformation is still a 4:1 impedance transformation, but that each line has a slightly different transformation ratio, then a very good match is obtained over a 40% bandwidth. In figure 2, the red curve corresponds to the impedance transformation ratio being optimised to obtain close to a 50 ohm match over as wide frequency range possible. The first line has $R_1 = 0.6883$, corresponding to a characteristic impedance of 34.4 $\Omega$ and the second line has $R_2 = 0.7264$ corresponding to a characteristic impedance of 17.2 $\Omega$. Applying similar principles to a 3-line section gives an even wider bandwidth, as shown by the brown curve in figure 2.
Wilkinson Transmission Line Hybrid

The Wilkinson transmission line hybrid consists of two quarter-wavelength long transmission lines forming a combiner, together with a load resistor to provide isolation. The circuit is shown in figure 3 and the performance is shown in figure 4.

For the analysis, an input is applied to port 2. For proper hybrid operation, no voltage should appear at Port 3. Transmission line TL2 has a standing wave on it with 0 Volt at the Port 3 side. Since the line is one-quarter wave long, there will be no current flowing into the line at the Port 1 side, as shown in figure 5. For the analysis of the voltage on port 1, we can thus remove TL2, since it is an open circuit at the port 1 end. The voltage at port 1 will thus simply be what we would have if one had the transmission line only, as shown in figure 6.
Since the input power is the same as the output power, the voltage $V_c$ at port 1 and the voltage $V_a$ at port 2 relate as $V_a = \sqrt{2} V_c$.

The maximum current and maximum voltage on a transmission line relate as:

$$V_{\text{max}} = Z_0 I_{\text{max}}$$

Eqn. 7

In figure 3, the current at the port 3 side of TL2 will thus be:

$$I_{\text{max}} = \frac{V_c}{\sqrt{2}Z_0} = \frac{V_a}{\sqrt{2}\sqrt{2}Z_0} = \frac{V_a}{2Z_0}$$

Eqn. 8

Since this current must come through the load resistor $R$ then

$$I_{\text{max}} = \frac{V_a}{R} = \frac{V_a}{2Z_0}$$

Eqn. 9

So that $R = 2Z_0$ for best isolation.

The circuit can be realised using a microstrip PCB layout. Figure 7 shows a typical realisation. A surface mount resistor is used. To ensure that there is no coupling between each of the arms of the hybrid, the spacing cannot be made too small and small
transmission lines are required to connect the resistor into the circuit. The characteristic impedance of those lines must be such that the resistor can be soldered to it. These short lines will have some effect on the performance as shown in figure 8, where the notch of $S_{11}$ is shifted in frequency as a result. The performance is however still satisfactory, since a 20 dB isolation or return loss is normally sufficient.

Figure 7. PCB layout of a Wilkinson Hybrid.

Figure 8. Performance of stripline Wilkinson Hybrid of figure 7.

**Compensated Wilkinson Hybrid**
In a Compensated Wilkinson Hybrid, a two step impedance transformation is used, like the two line impedance transformation of figures 1 and 2. The resulting circuit is shown in figure 9. The impedance transformation from line TL3 is from $50 \, \Omega$ at port 1 to $35.35 \, \Omega$ at the right of TL3. Lines TL1 and TL2 do an impedance transformation from $50 \, \Omega$ at port 2 and port 3 to $70.7 \, \Omega$ at the left hand side of transmission lines TL1 and TL2. Those in parallel give a $35.35 \, \Omega$, which matches that at the right hand side of line TL3. Each of the three transmission lines will thus have the same impedance transformation ratio of $\sqrt{2}$.

Figure 10 shows a comparison between the conventional, shown in red, and the compensated (blue) Wilkinson Hybrid. The Compensated Wilkinson Hybrid shown in blue has a wider frequency response for both the isolation and coupling. In addition the compensated Wilkinson Hybrid uses lower impedance transmission lines, resulting in wider track widths for the microstrip layout, thus lowering the resistive losses in the circuit.

**Unequal Split Wilkinson Hybrid**

Sometimes an unequal power split is required. If a power ratio of $P:1$ is required, then the impedances seen at the source end of each of the transmission lines must also have a ratio of $P:1$. Since the total impedance must be $Z_0 = 50 \, \Omega$, then the impedances are:

$$\frac{1}{Z_0} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_1} + \frac{1}{P*Z_1} = \frac{P+1}{P*Z_1}$$

Eqn. 10.

The line impedances are thus:

$$Z_1 = \sqrt{\frac{P+1}{P}} Z_0$$

Eqn. 11

$$Z_2 = \sqrt{P+1} * Z_0$$

Eqn. 12
where $Z_1$ and $Z_2$ are the transmission line impedances of the hybrid. As a check when an equal split is used, $P=1$, so that $Z_1=Z_2=\sqrt{2}Z_0$.

Thus for a 10:1 power split in a 50 Ω system, the transmission line impedances are:

$Z_1=\sqrt{1.1} Z_0$ and $Z_2=\sqrt{11} Z_0$. For a 50 Ω system, $Z_2 = 165 \Omega$. For an RF circuit board substrate, RO4003, which is 0.818 mm thick, at 1 GHz a 50 Ω track is 1.855 mm wide and a 165 Ω track is 0.063 mm wide. The 165 Ω track is too thin to make accurately. The track width limits the ratio of the power split. Lowering the impedances of the split lines by reducing the impedance at the junction point just like the compensated Wilkinson hybrid, will reduce the bandwidth of the hybrid but will permit it to be constructed. Increasing the substrate thickness will also increase the track width.

**Wideband Wilkinson Hybrid**

The bandwidth of the Wilkinson hybrid can be increased by cascading two resistor linked transmission lines as shown in figure 11. The Characteristic impedances of the transmission lines and the resistor values can be optimised to give the required isolation over the specified bandwidth. There is a compromise between bandwidth and isolation. A wide bandwidth will result in a low isolation and a smaller bandwidth can result in a better isolation. For the hybrid shown in figure 11 the isolation was specified as -30 dB.
From Figure 12, it can be seen that a wide bandwidth hybrid results. The impedance for the first section has increased to 83Ω compared with 70.7Ω for the conventional hybrid. The higher impedance will result in an increased insertion loss of the hybrid, due to thinner PCB tracks being required. A compensated hybrid configuration will reduce the impedance values and thus reduce the insertion loss of the hybrid as well as having a wider bandwidth still.

This process can be extended to produce Wilkinson hybrids with good isolation and return loss on all ports over a 2:1 or even 3:1 frequency range. Figure 13 shows the schematic of a 3:1 Wilkinson Hybrid covering the frequency range from 90 MHz to 270 MHz. The values or transmission line impedances and resistor values are those obtained after optimisation. The target specification is a better than 30 dB isolation and better than 30 dB return loss for all the ports. This circuit achieves those specifications.

The design process requires two steps. Firstly the circuit is designed and optimised using ideal transmission lines for the hybrid, as shown in Figure 13. Once the
specifications are achieved for this ideal circuit, then the ideal transmission lines are replaced with microstrip lines, with the same electrical line length and impedance.

Often it is necessary to bend the microstrip lines to fit the circuit in the available space, as shown in figure 14. Each of the transmission lines are a quarter wavelength long. If in figure 13 lines TL2 and TL5 are disabled, resistors R1 and R3 are disabled and resistor R2 is made a short circuit (0 \( \Omega \)), then the circuit becomes a transmission line of various impedances connecting port 1 to port 2 with an open circuited quarter wavelength stub consisting of TL7 connected to it. This line will thus present itself as a short circuit at R2 at the centre frequency. The transfer function from ports 1 to 2 will thus have a notch exactly at the centre frequency of the hybrid, where TL7 is a quarter wavelength. The length of the folded sections making up TL7 in figure 13, can be adjusted to be exactly a quarter wavelength by changing the lengths of the sections to ensure that the notch in the transfer function from ports 1 to 2 occurs exactly at the centre frequency of the hybrid. This process is repeated for the other three different transmission lines in the hybrid. Variables are used to ensure that the folded transmission lines corresponding to the sets of lines TL1 and TL2, lines TL3 and TL5 and lines TL6 and TL7 are exactly the same length and impedances. The accompanying MWO example file can be used to further illustrate this process.

Finally the line lengths and widths are optimised to provide the fine tuning required for the final layout to again meet the specifications. Figure 14 shows the hardware for this layout and figure 15 shows the simulated frequency response for the hybrid. The measured performance agrees closely with the calculated one and the hardware has an isolation that is better than 28 dB over the entire 90 to 270 MHz frequency range and a similar return loss on all ports. The slight degradation in performance is due to slight imperfections in the hardware compared with the simulation.
The losses in the circuit due to the resistance of the tracks, dielectric losses of the substrate and radiation losses of the microstrip lines, varies with frequency, as can be seen in figure 15. However since the difference is only 0.25 dB over a 3:1 frequency range and can be ignored in practice. Due to this frequency dependence of the losses, it is better to optimise for input reflection coefficient rather than transfer function since a low input reflection coefficient ensures all the energy passes through the hybrid, thus giving the highest $S_{21}$ possible. Optimising for a value of $S_{21}$ may result in a poor return loss at lower frequencies.

### Quarter Wave Hybrid or 1.5 $\lambda$ Rat-race Hybrid

The circuit diagram of this hybrid is shown in figure 16 and it’s performance is shown in figure 17. For the analysis consider the function of the hybrid. If an input is applied to port 1, then no signal should appear at port 3, resulting at 0 Volt at port 3. Under these conditions, the circuit is similar the Wilkinson hybrid of figures 5 and 6, and no current flows into TL3 of figure 16 at port 2 and into TL4 at port 4. The transmission line TL3 can thus be disconnected at port 4 and line TL4 can be disconnected at port 2, without any effect, as shown in figure 18.
The resulting circuit is thus just like a Wilkinson Hybrid, without the terminating resistor. The line impedances should thus be 70.7 Ω, just like the Wilkinson Hybrid. The same analysis can be repeated by looking into each port in turn. A three-quarter wavelength transmission line has exactly the same impedance transformation ratio as a quarter wavelength line. All the line impedances will thus be \( \sqrt{2}Z_0 \).

The 1.5 \( \lambda \) Rat-Race Hybrid is commonly used in high power transmitters, and has an advantage over the Wilkinson Hybrid that for a 50 Ω system, a 50 Ω load resistor to ground is required for the termination rather than a 100Ω load resistor between two active inputs. High power 50 Ω loads are readily available from manufacturers such as Bird Electronic Corporation. 100 Ω floating loads are however more difficult to obtain.
Figure 19 shows an RF PCB realisation of a 1.5 λ Rat-race Hybrid. Apart from a slight (0.25 dB typically) increase in insertion loss due to the resistive losses in the microstrip tracks, the performance of the PCB layout of the 1.5 λ Rat-race Hybrid is the same as shown in figure 17.

Lower line impedances can be obtained with a compensated 1.5 λ Rat-race Hybrid, by having 44.5 Ω quarter wavelength transmission lines at the 4 inputs to the hybrid, for a 50 Ω system. The line impedances of the hybrid then become 56.119 Ω instead of 70.7 Ω. Computer simulation shows that this does not increase the bandwidth of the isolation and it does not reduce the losses for a typical PCB layout, so that compensated 1.5 λ Rat-race Hybrid are not used in practice.

Branchline Coupler
The Branchline coupler consists of 4 transmission lines in a ring, as shown in figures 20 and 21. If an input is applied at the top left port 1, then part of the output appears at the coupled port, the bottom right port 3. The remainder appears at the main output port 2 and no power appears at the isolated port 4. For a 3 dB coupler in a 50 Ω system, lines TL1 and TL4 have an impedance of $50/\sqrt{2}$ Ω and lines TL2 and TL3 have an impedance of 50 Ω. A Microstrip PCB layout is shown in figure 21. The resistive losses of a Branchlike coupler are less than that of a 1.5 λ Rat-race Hybrid.

For the analysis, consider figures 20 and 22. When an input is applied at port 1, port 4 is isolated and has no voltage at that port. Since lines TL3 and TL4 are quarter wavelength long, no current flows in line TL4 at port 3, so that TL4 can be disconnected from the circuit without any effect. Similarly, no current flows in TL3 at port 1, so that TL3 can also be disconnected, resulting in figure 22.

If a normalised input power of 1 is applied at port 1 and we want an output power of $P$ to occur at port 3, then a power of $(1-P)$ will be available at port 2.
At port 2, the load impedance is $Z_0$, which typically is $50 \, \Omega$. If the voltage at port 2 is $V_2$, then the power split at port 2 is given by:

$$V_2^2 = PZ_P = (1 - P)Z_0$$

so that:

$$Z_P = \frac{(1 - P)Z_0}{P}$$

Eqn. 13.

Where $Z_P$ is the impedance of port 3 transformed through TL1 and seen at port 2 looking into TL1. Since this is obtained by impedance transformation through the quarter wave long transmission line TL1, the impedance required for TL1 is given by:

$$Z_Y = \sqrt{Z_PZ_0} = Z_0\sqrt{\frac{(1 - P)}{P}}$$

Eqn. 14

The impedance $Z_2$ seen at the end of TL2 is $Z_0$ in parallel with $Z_P$ and is thus:

$$\frac{1}{Z_2} = \frac{1}{Z_P} + \frac{1}{Z_0} = \frac{P}{(1 - P)Z_0} + \frac{1}{Z_0} = \frac{1}{(1 - P)Z_0}$$

Eqn. 15

The line impedance $Z_X$ to transform this to $Z_0$ is thus:

$$Z_X = Z_0\sqrt{(1 - P)}$$

Eqn. 16

In many cases an equal power split is required, so that $P=0.5$. Typically $Z_0 = 50 \, \Omega$. Substituting this in equations 14 and 16 results in $Z_Y = Z_0 = 50 \, \Omega$ and $Z_X = 0.707*Z_0 = 35.35 \, \Omega$. For a 10 dB coupler $P = 0.1$, so that $Z_Y = 3Z_0 = 150 \, \Omega$ and $Z_X = 0.949*Z_0 = 47.43 \, \Omega$. These can just be made using microstrip circuits, however any lower amount of coupling is extremely difficult to make.

Figure 23 shows the performance of the Branchline coupler. This coupler is a convenient structure with low impedance values ($50 \, \Omega$ and $35.35 \, \Omega$) thus allowing low loss hybrids to be made using microstrip circuits.

Figure 23. Branchline coupler performance.
In many cases the transmission lines are folded in order to reduce the printed circuit board area, as shown in figure 24. The losses in the microstrip circuit causes a reduced isolation at port 4 and results in a slightly asymmetrical coupling as shown in figure 25. The performance is however very similar to that of figure 23.
The output at the coupled port has a 90° phase shift compared with the direct port and that is useful for many applications. Figure 26 compares the phase difference of signals at port 2 and 3 due to an input at port 1 and compares this with the phase shift in a simple transmission line. It can be seen that at the centre frequency of the hybrid there is only ±1.22° variation in phase for a ±10% variation in frequency, while it is ±9° for a single transmission line. The Branchline coupler can thus be used to provide the I and Q signals, required for image suppressing mixers, with good phase and amplitude accuracy over a 20% bandwidth.

It is possible to produce a wider bandwidth Branchline coupler, by having two loops together as shown in figure 27. For a 3 dB coupler the characteristic impedances of the lines can be shown to be 120.7 Ω for the outer vertical lines, 60.35 Ω for the centre line and 46.2 Ω for the horizontal lines. Figure 27 also shows the corresponding PCB layout of the circuit. For a RO4003 RF PCB substrate with a 0.818 mm thickness at 1 GHz, the 120.7 Ω transmission lines require 0.2387 mm wide tracks, which is close to the smallest track width that can be realised. These tracks are thus very difficult to achieve in practice. It is possible to increase the bandwidth further by using three loops for the coupler. The outer track impedances are then 175 Ω. For this RO4003 substrate this corresponds to a 0.04 mm wide track, which is not possible to realise. Doubling the substrate thickness doubles the track width for the same impedance. Using thicker substrates may make some of these configurations realisable.
Figure 27. Wideband Quadrature Hybrid Schematic and the corresponding PCB layout.

Figure 28 shows a comparison of the single, double and triple loop Branchline coupler. The green curves are the single line coupler, the blue curves are the double line coupler and the red curves are the triple line coupler. The most significant difference is the isolation (between port 1 and 4). The bandwidth over which a good isolation is obtained is increased significantly by using multiple loops.
Backward Travelling Wave Hybrid

If two lines are in close proximity, then there is coupling between the lines. The amount of coupling depends on the spacing between the lines. TxLine in Microwave Office can be used to calculate the coupling between lines of a given spacing, as shown in figure 29.

Consider the two microstrip coupled lines as shown in figures 29 and 30. The maximum coupling will occur when the coupled lines are a quarter wavelength long. The resulting coupling is related to the even and odd mode impedances by:

\[
Z_{OE} = Z_0 \sqrt{\frac{1 + C}{1 - C}} \quad \text{Even mode impedance.} \quad \text{Eqn. 13}
\]

\[
Z_{OO} = Z_0 \sqrt{\frac{1 - C}{1 + C}} \quad \text{Odd mode impedance.} \quad \text{Eqn. 14}
\]
Where C is the voltage at the coupled output relative to the input, $Z_{OO}$ is the odd mode impedance, where the two lines of the transmission line are of opposite polarity, so that most of the field is between the conductors. $Z_{OE}$ is the even mode impedance, where both the lines are at the same potential, so that most of the field is between the conductors and the ground-plane.

**Edge Coupled Lines**

Edge coupled lines correspond to the geometry shown in the top right hand side of figure 29. The coupled lines are made using conventional microstrip lines using double sided PCB with the bottom side a ground-plane. This is easy to manufacture, but places restrictions on the amount of coupling that can be obtained. Typically the spacing between the lines is very small and this spacing must be accurately controlled to obtain the correct amount of coupling. Edge coupled lines are very suitable for sampling the forward and reverse power from transmitters and for filters. The design process of a backward coupled hybrid using edge coupled lines is illustrated with the following example.

**Example:** A 20 dB coupler for a transmitter at 900 MHz is required. The coupled output is thus 20 dB below the input. The coupled voltage is one tenth of the input voltage, i.e. C = 0.1. From the above equations one obtains $Z_{OO} = 45.23 \, \Omega$ and $Z_{OE} = 55.28 \, \Omega$. The required physical spacing can now be determined iteratively using the Txline program. The final results from the Txline calculations are shown in figure 29. Note that for the same line length, the electrical length is different for the even and odd mode, since the velocity of propagation of these modes is different. The length chosen is such that the average is correct, but it is likely that the length needs to be fine-tuned to obtain the desired performance.

![Figure 30. λ/4 20 dB coupler Schematic and Corresponding Microstrip Layout.](image-url)
These calculated values are substituted in the coupled line circuit diagram shown in figure 30. Figure 30 also shows the corresponding PCB layout including 50 Ω lines needed to connect the input and output ports to the coupler. Figure 31 shows the corresponding performance. The coupling is 20 dB as expected. Note that the coupled port is in a different location compared with the Branchline coupler.

The coupler was designed for a centre frequency of 1 GHz. Coupled lines have a null in coupling at multiples of a half wavelength length. This was used to tune the length of the coupled line to give a null at 2 GHz, thereby giving maximum coupling at the required 1 GHz. Note that the bandwidth of the coupling and the isolation is very wide compared with the other hybrids described before in these notes. At 1 GHz, the isolation (S_{31}) is only 5 dB less than the coupled output (S_{41}), so that this coupler cannot be used as a directional coupler, without further design changes.

![Microstrip Edge Coupler](image)

**Figure 31.** \(\lambda/4\) Microstrip 20 dB coupler performance.

Alternately, the coupling can be obtained directly from figure 30. Changing the length of the coupled lines, such that a coupling null is obtained at twice the frequency for maximum coupling, (2 GHz for figure 31) and the spacing of the coupled lines is changed to obtain the correct coupling factor and the coupled line with is changed to obtain a low return loss (S_{11}), allows the same coupler to be designed, without using the Txline program. These values can easily be determined using the optimisation routines included in MWO.

By including some very small, sub-pf, capacitors across the terminals 1 and 4 and terminals 2 and 3 the phase velocity is more equalised over a wide frequency range and a lower input return loss (S_{11}) and flatter isolation (S_{31}) is obtained.

One application of the backward travelling wave coupler is in the measurement of power at the output of a device, such as a signal generator or a transmitter. Typically a coupled signal –20 dB below the forward or reverse power is required. The forward power is the transmitter output power and the reverse power is the reflected signal that occurs if the load for the transmitter is mismatched. To accurately differentiate between forward and reflected power at least a 20 dB front to back isolation is required. The hybrid will thus require a coupled output of –20 dB and an isolated signal < –40 dB.
As can be seen from figure 31, the signal at the isolated port ($S_{31}$) is too large when a quarter wavelength long coupler is used. A suitable coupler with a good forward to reverse ratio can be obtained by making the length of the coupler smaller than a quarter wavelength and using capacitors shunting the forward and reverse coupled outputs to produce a constant frequency response over the required frequency band.

Figure 32 shows the simulated performance of such a 20 dB coupler. Comparing this with figure 31 shows that the isolated port has a much better isolation and the coupled port has a flatter and wider frequency response. This performance is obtained by using the optimiser in Microwave Office to vary the coupling gap, line widths, lengths and capacitance values until a suitable performance is obtained. The resulting PCB layout is shown in figure 33. For a $\lambda/4$ long coupler and $-20$ dB coupling, a 0.95 mm gap is required. For the shorter length coupler of Figure 33, the coupling gap is 0.2 mm, which is easy to achieve using current technology. Figure 34 shows a photograph of the
stripline coupler. The measured performance closely matches the simulated performance. Figure 35 shows the corresponding circuit diagram. The blue variables for widths, length, coupling gap and capacitor values are the parameters that are optimised to obtain the specified performance.

Figure 34. Photograph of the Microstrip-line coupler.

Figure 35. Circuit diagram of Microstrip-line coupler.

For this application Edge coupled Backward travelling wave hybrids are the best choice since a wide bandwidth and low coupled outputs (-20 dB) are required.
**Lange Coupler**

For a 3 dB coupler, $C=0.707$ and $Z_{OE}=120.7\ \Omega$ and $Z_{OO}=20.7\ \Omega$. For edge coupled microstrip lines, this requires a coupling gap far smaller than 0.1 mm, which is impossible to make. To increase the coupling, many lines can be used in parallel. This is called a Lange Coupler. Figure 36 shows a layout of a Lange coupler. At the middle a bonding wire is used to connect the upper track end to the middle track and the lower other track end, thus keeping those transmission lines in parallel. Similarly the track ends at the edges of the coupler are connected to the other appropriate lines. One will thus have the top left port connected by a set of parallel lines to the bottom right port. The top right port is connected by a set of parallel lines to the bottom left port. Typically the spacing between the tracks and the track widths are of the order of 0.25 mm, which is the smallest widths that can conveniently be made using conventional processes.

![Lange coupler layout](image)

Even having these parallel lines result in very fine track widths and coupling gaps, making the Lange Coupler expensive to make. The Lange coupler is however the only planar hybrid that offers better than an octave (2:1) of useable bandwidth. With the high cost of manufacturing the Lange Coupler, due to the small track and gap widths and the high cost of attaching the bond wires, Lange Couplers are not often used. The required coupling can be obtained at a lower cost and can be better controlled using broadside coupled lines and multilayer RF printed Circuit Boards. They can also be implemented with Low Temperature Co-fired Ceramic (LTCC) substrates which are discussed in chapter 10.

**Broadside coupled lines**

It is possible to have multilayer printed circuit boards with RF substrates. These can be used to design coupling structures with higher coupling and 3 dB couplers are quite practical. The typical configuration is shown in figure 37.

The advantages of this structure are firstly that bigger coupling between the transmission lines can be obtained with a good control over impedance values. Secondly the stripline structure has no radiation losses. The disadvantage is that a non-planar structure is required.
Figure 37. Stripline broadside coupled lines

Txline is not capable of calculation the broadside coupled transmission line coupling gap (s) and line width (W), shown in figure 37. The Agilent’s Linecalx program does allow this. In addition books like Matthei, Young and Jones. Microwave Filters, Impedance Matching Networks and Coupling Structures. Artech House 1980. (McGraw Hill 1964) pages 180 and 181 give empirical equations for $Z_{oo}$ and $Z_{oe}$ are given in equations 15 and 16. The capacitance $C_{fe}$ and $C_{fo}$ are fringing capacitances between the coupled lines and the ground plane and these can be determined from figure 38.

\[
Z_{OE} = \frac{188.3}{\sqrt{\varepsilon}} \left( \frac{w/b - C_{fe}}{1 - s/b} + \frac{1}{\varepsilon} \right)
\]

Eqn. 15

\[
Z_{OO} = \frac{188.3}{\sqrt{\varepsilon}} \left( \frac{w/b + w + C_{fo}}{1 - s/b} + \frac{1}{s \varepsilon} \right)
\]

Eqn. 16

Figure 38. Table for $C_{fe}$ and $C_{fo}$ from Matthei, Young and Jones, pp181.
Using the two-line broadside coupled line element, SBCPL, and the schematic circuit simulation, allows an easier and more accurate determination of the broadside coupler parameters, than using equations 15, 16 and figure 38. The SBCPL element is a two-line broadside coupled line element. The cross-section is similar to that of Figure 37.

In addition the width of each of the lines can be specified independently, the lines can be offset and the dielectric constants between the lines can be different from that above and below the line. None of these advanced features can be determined using the alternative techniques. The use of the SBCPL element to determine the required coupling gap and line width for a coupler is illustrated in the following example.

Example

A 3 dB coupler was required to operate at a power level of about 100 Watt at a frequency of 100 MHz. At the time of the design, MWO did not yet exist and the Equations 15 and 16 and Figure 38 were required to complete the design.

Two 12.6 mm thick pieces of Polyethylene, which has a dielectric constant of $\varepsilon_r = 2.26$, were used for the top and bottom dielectric sheets for this coupler. Polyethylene sheets of this thickness can conveniently be purchased as a cutting board for bread from most local supermarkets. For a 3dB coupler, $C = 0.707$ and $Z_{OE} = 120.7 \, \Omega$ and $Z_{OO} = 20.7 \, \Omega$. Since these impedances must be evaluated iteratively, it is the easiest to enter the above equations for $Z_{OE}$ and $Z_{OO}$ into an Excel or other spreadsheet. Using the values $B = 29$ mm, $s = 2.9$ mm, $w = 12.7$ mm, gives $\frac{s}{B} = 0.1$ and thus from the table gives $\frac{C_1}{\varepsilon} = 0.56$ and $\frac{C_2}{\varepsilon} = 1.15$. This then results in $Z_{OE} = 119.7 \, \Omega$ and $Z_{OO} = 20.8 \, \Omega$, which is close enough. At 100 MHz the length of a quarter wavelength line is given by: $\frac{\lambda}{4} = \frac{c}{\sqrt{\varepsilon F}} = 50 cm$. As shown in figure 39, the width of 50 $\Omega$ terminating lines can be calculated using Txline or equivalent programmes as 23.2 mm. A 2.9 mm thickness of Polyethylene is not readily available, however a 3.17 mm thickness of Perspex, with a dielectric constant of approximately 3.42 is available and was used. The effect of this substitution cannot readily be calculated using the above techniques. This coupler was constructed and performed as expected, even though the line spacing was not ideal.

![Figure 39. Terminating line-width calculation.](image)

The use of SBCPL in Microwave Office gives much more flexibility and allows a more accurate determination of the performance. The actual cutting boards used are 12.6 mm
(0.5 inch) thick, pieces of Polyethylene. They are to be used for the top and bottom dielectric and a 3.17 mm (0.125 inch) thick Perspex sheet is available to separate the tracks. Entering these values in the SBCPL element and then fine-tuning the track width, track length and offset, allows the required performance to be obtained. Polyethylene has a dielectric constant of $\varepsilon_r = 2.26$ and Perspex has a dielectric constant of $3.42 \pm 0.3$.

Tuning the coupler length ($L = 449$) sets the centre frequency of the coupler to 100 MHz and tuning the line width ($W = 12.2$) adjusts the coupling to have close to 3 dB coupling over the 85 MHz to 115 MHz frequency range and achieve a return loss on the
ports of less than -20 dB. The resulting MWO circuit diagram is shown in figure 40 and the corresponding performance is shown in figure 41. The track width of 12.2 mm and length of 440 mm obtained from MWO agrees closely with the 12.7 mm and 500 mm obtained for the earlier design using the equations 16 and 17 and figure 38 and used for the construction of the coupler. The MWO design is far more accurate since it allows for losses in the dielectric and the Perspex sheet for separating the tracks, having a different dielectric constant and loss tangents compared with the Polyethylene.

*Prepreg* sheets are available, which can be used to convert single layer PCB’s to multilayer PCB’s, so that broadside couplers can be constructed. The Roger’s RO4403 prepreg sheets have a dielectric constant of 3.17 and are 0.1 mm thick and can be used to bond two single layer PCB’s to form a multilayer PCB. The sandwich is pressed together with a pressure of 2.8 MPa (400 psi) and a temperature controlled profile up to 180 °C (350 °F) for a cycle time of 2 hours to form the correct bond. For further details see the RO4403/4450 data sheet. Other manufacturers have similar materials.