Chapter 8

Amplifiers: Stability, Noise and Gain

IF amplifiers

A few years ago, if one had to use an RF or IF amplifier, one had to design that using individual transistors and associated passive components. Now for many commercial RF frequency bands there are low cost IC’s available which perform as well or better than discrete transistor designs. For example the RF-Micro Devices RF2489 IC has a low noise amplifier covering the 869 to 894 MHz mobile radio bands with a noise figure of 1.1 dB, which is better than the transistor amplifier example used in these lectures. That IC also has a mixer, resulting in a low cost system. The IC’s also have less stability problems. If a low noise amplifier, without a mixer is required, then the RF Micro Devices RF2361 amplifier can be used and that has a noise figure of 1.4 typically at 881 MHz and 1.3 at 1.95 GHz. Many other manufacturers have similar devices.

For IF amplifiers at HF frequencies it is now also possible to use high speed operational amplifiers, resulting in IF strips that have an excellent linearity and very low intermodulation distortion. For medium power amplification, VHF cable TV repeaters can be used. There are also many amplifier modules available with a very high linearity, particularly for frequency bands for mobile phones, wireless LAN, Bluetooth and other high volume consumer frequencies.

MMIC

Several IC manufacturers like Agilent, Minicircuits and Freescale (Motorola) produce general purpose Microwave Monolithic Integrated Circuits (MMIC): These can be used as general purpose amplifier blocks and can be used at frequencies from DC up to 10GHz. Several devices also have low noise figures, for example, the Minicircuits MAR6 amplifier has a bandwidth of DC to 2 GHz and a noise figure of 3 dB and the GALIS66 amplifier has a bandwidth of DC to 2 GHz and a noise figure of 2.7 dB. These devices have sufficient low noise figures to result in economical commercial solutions.

Figure 1. Typical Minicircuits MMIC circuit diagram.
There are basically two types of general purpose MMIC’s: 1) Fixed voltage and 2) Constant current. The fixed voltage MMIC’s are designed to operate at a supply voltage of typically 5 or 3.6 Volt.

Minicircuit’s MAR, ERA, Gali and Lee series of amplifiers all require a constant current source and have a typical circuit diagram as shown in figure 1. Many of the MMIC’s are made using InGaP HBTs (indium gallium phosphide heterojunction bipolar transistors). Others are made using gallium arsenide Enhancement Mode Pseudomorphic High Electron Mobility Transistor (PHEMT).

A suitable constant current supply is achieved by using a resistor to set the current supplied to the MMIC. The RF loading of that resistor is minimised by having a large inductor in series with the resistor. Since the resistor is typically $600 \, \Omega$ for a 12 Volt supply voltage, in many applications a slightly reduced gain can be tolerated and the inductor can be removed. This series resistor, together with the DC feedback resistor, between the collector and base of the transistor in figure 1, result in a stable quiescent current as the temperature changes.

Most of the devices are unconditionally stable. However since these devices have very wide bandwidths, microwave layout techniques must be used to ensure that the devices remain stable. As a typical example it is essential that the inductance of the earth pin connections of the MMIC is kept as low as possible. This is achieved by ensuring that earth pins of the MMIC are soldered to a large ground plane and that very close to the earth pin of the MMIC that ground-plane has a via connecting the top ground-plane to the bottom ground-plane of the microstrip PCB layout.

These amplifier modules and MMIC’s can now be used for the majority of RF and microwave designs. There are however some applications, such as very low noise amplifiers of very high frequency applications and high power amplifiers, where amplifier modules and MMIC’s can not be used. For such designs a knowledge of stability, noise and gain circles of the amplifier is essential.

**Stability**

For power amplifier design single power transistors are used to provide the required amplification at the required frequency, with the required linearity. The amplifier should be stable and not oscillate at any frequency under normal operating conditions. Amplifier stability and ensuring that the transistor is matched, so that the highest power output is obtained are the most critical parameters in a power amplifier design.
**Smith Chart Revision**

A good reference for this material is D. M. Pozar: “Microwave Engineering” 3rd Edition, Wiley, Chapter 11

![Smith Chart](image)

Figure 3 shows a Smith Chart, which is a plot of the normalised reflection coefficient $\Gamma$.

$$\Gamma = \frac{Z_L - 1}{Z_L + 1} = |\Gamma|e^{i\phi} = \Gamma_r + j\Gamma_i$$

Hence the Smith chart also represents $Z_L$ and we can plot contours of $Z_L = R_L + jX_L$. Solving for $R_L$ and $X_L$ gives:

$$R_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} \quad \text{and} \quad X_L = \frac{2\Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$R_L$ and $X_L$ form a family of circles.

The Smith chart is a very useful tool for showing impedance values versus frequency.
Scattering Parameters Revision

![Diagram of 2 Port Network]

Figure 4. S parameter definitions

\[
\begin{bmatrix}
V_1^- \\
V_2^- \\
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22} \\
\end{bmatrix}
\begin{bmatrix}
V_1^+ \\
V_2^+ \\
\end{bmatrix}
\]

- \(S_{11} = \frac{V_1^-}{V_1^+} \) \(V_2^+ = 0\) i.e. reflection coefficient at port 1 with an ideal match at port 2
- \(S_{22} = \frac{V_2^-}{V_2^+} \) \(V_1^+ = 0\) i.e. reflection coefficient at port 2 with an ideal match at port 1
- \(S_{21} = \frac{V_2^-}{V_1^+} \) \(V_2^+ = 0\) i.e forward gain
- \(S_{12} = \frac{V_1^-}{V_2^+} \) \(V_1^+ = 0\) i.e reverse gain, i.e effect of output on input.

An amplifier is unilateral if \(S_{12} = 0\) i.e. the reverse gain is zero. That applies to many amplifiers, particularly operational amplifiers and other IC’s where the output is well separated from the input.

The input reflection coefficient is:

\[
\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_{Load}}{1 - S_{22} \Gamma_{Load}} \quad \text{For a unilateral amplifier } \Gamma_{in} = S_{11}
\]

\[
\Gamma_{out} = S_{22} + \frac{S_{21} S_{12} \Gamma_{Source}}{1 - S_{11} \Gamma_{Source}} \quad \text{For a unilateral amplifier } \Gamma_{out} = S_{22}
\]

The available power gain of an amplifier is:

\[
G_A = \frac{\text{Power available from network}}{\text{Power available from source}} = \frac{|s_{21}|^2 (1 - |\Gamma_S|^2)}{\left|1 - s_{11} \Gamma_S (1 - \left|\Gamma_{out}\right|^2)\right|} \quad \text{(Pozar 3rd ed. Eqn. 11.12)}
\]

The Available Power Gain is a useful measure of how well the amplifier is matched.

\[
G_T = \frac{\text{Power delivered to the load}}{\text{Power available from source}} = \frac{|s_{21}|^2 (1 - |\Gamma_S|^2) (1 - |\Gamma_L|^2)}{\left|1 - \Gamma_S \Gamma_{in} (1 - \left|\Gamma_{out}\right|^2)\right|} \quad \text{(Pozar 3rd ed. Eqn. 11.13)}
\]

The Transducer Power Gain is a useful measure of the gain of the amplifier. There are measurement functions for available power gain and transducer gain in Microwave Office.
Under ideal conditions, \( \Gamma_s = 0 \) and \( \Gamma_L = 0 \) so that \( G_d = G_r = |S_{21}|^2 \).

An amplifier includes input and output matching circuits as shown in Figure 5. Amplifier Block Diagram.

The input match provides maximum power when \( \Gamma_{\text{in}} = \Gamma_{\text{source}}^* \) so that the gain of the input matching network is \( G_S = 1 \). Similar when the output has a conjugate match so that \( \Gamma_{\text{out}} = \Gamma_{\text{load}}^* \), then \( G_L = 1 \). In general:

\[
G_S = \frac{1 - |\Gamma_{\text{source}}|^2}{1 - \Gamma_{\text{in}} \Gamma_s} \quad \text{(Pozar 3rd ed. Eqn. 11.17a)}
\]

\[
G_\theta = |S_{21}|^2 \quad \text{(Pozar 3rd ed. Eqn. 11.17b)}
\]

\[
G_L = \frac{1 - |\Gamma_{\text{load}}|^2}{1 - \Gamma_{\text{out}} \Gamma_L} \quad \text{(Pozar 3rd ed. Eqn. 11.17c)}
\]

The gain of the whole amplifier is:

\[
G_r = G_d * G_\theta * G_L
\]

Under ideally matched conditions \( G_S = 1 \) and \( G_L = 1 \) so that the gain is thus:

\[
G_r = |S_{21}|^2, \text{ as was obtained before.}
\]

**Stability Requirements**

Oscillations will occur if \( G_S \rightarrow \infty \) or \( G_L \rightarrow \infty \).

For oscillations due to the input, \( \Gamma_{\text{in}} * \Gamma_S = 1 \). If \( \Gamma_S = 1 \) then for oscillations \( \Gamma_{\text{in}} = 1 \).

\[
\Gamma_{\text{in}} = S_{11} + \frac{S_{12} S_{21} \Gamma_{\text{Load}}}{1 - S_{22} \Gamma_{\text{Load}}}
\]

This is a circle on the Smith Chart called Input Stability Circle. Oscillators often are designed by deliberately making \( \Gamma_{\text{in}} * \Gamma_S = 1 \) and having the source impedance a frequency selective network, such that this condition for oscillation is only satisfied at one frequency. The amplifier is stable for input conditions if \( \Gamma_{\text{in}} < 1 \).

For oscillations due to the output \( \Gamma_{\text{out}} * \Gamma_L = 1 \). If \( \Gamma_L = 1 \) then for oscillations \( \Gamma_{\text{out}} = 1 \).

\[
\Gamma_{\text{out}} = S_{22} + \frac{S_{21} S_{12} \Gamma_{\text{Source}}}{1 - S_{11} \Gamma_{\text{Source}}}
\]
This is a circle on the Smith Chart called Output Stability Circle. The amplifier is stable for output conditions if $\Gamma_{\text{out}} < 1$. Microwave Office allows Stability Circles to be plotted at different operating conditions. To see these circles the region outside the normal Smith Chart may need to be viewed.

For unilateral amplifiers $S_{12} = 0$, so that $\Gamma_{\text{in}} = S_{11}$, and $\Gamma_{\text{out}} = S_{22}$. This makes it relatively easy to ensure that the amplifier is stable.

**Unconditional Stability**

An amplifier is unconditionally stable if $|\Gamma_{\text{in}}| < 1$ and $|\Gamma_{\text{out}}| < 1$ for all passive load and source impedances. As an example, the first version of the MAR6 amplifier is unconditionally stable. For this to occur, both the input and output stability circles lie outside the normal Smith Chart. The second version, the MAR6A amplifier is conditionally stable.

**Conditional Stability**

An amplifier is conditionally stable if $|\Gamma_{\text{in}}| < 1$ and $|\Gamma_{\text{out}}| < 1$ only for certain load and source impedances. As an example the early version of the MAR8 amplifier was only conditionally stable and both input and output reflection coefficients had to be less than 0.5. For conditional stability, either or both the input and output stability circles partially lie inside the normal Smith Chart.

**Stability Factors: Measures of Stability**

The Rollett’s stability factor, $K$ is defined as:

$$
K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2}{2|S_{11}S_{21}|}
$$

Where $|\Delta| = |S_{11}S_{22} - S_{12}S_{21}|$

In addition the auxiliary stability factor is defined as:

$$
B = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2
$$

For unconditional stability $K > 1$ and $B > 0$.

The Geometric Stability Factor, $\mu$ is defined as:

$$
\mu = \frac{1 - |S_{22}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|}
$$

This stability factor measures the distance from the centre of the Smith Chart to the nearest unstable region. For unconditional stability $\mu > 1$. In addition, the larger the value of $\mu$, the more stable the amplifier.

The Rollett’s stability factors $K$ and $B$ and the Geometric Stability Factors for both the input and output of an amplifier are available as measurements in Microwave Office, so that an amplifier’s stability can easily be determined at any frequency. It is important that the stability of an amplifier is determined at all possible frequencies, since if the device is unstable it will likely oscillate. If the designer only measures the performance of the amplifier in the operating band and oscillating frequency is far removed from this
frequency, the designer will be at a loss to explain the very poor amplifier performance until a spectrum analyser is used to check for oscillations over a wide frequency range.

If the amplifier is not unconditionally stable then the source and load must be carefully controlled to ensure that the amplifier is stable under normal operating conditions.

**Design for Maximum Gain**

To design an amplifier for maximum gain, the input matching network and the output matching network are both designed to provide a conjugate impedance match at the operating frequencies, thus making $G_S = 1$ and $G_L = 1$, as outlined above. Since the power gain depends on the impedance match at the input, one can plot constant gain circles on the Smith Chart, to show how the gain varies with the matching impedance. Designing an amplifier for maximum power gain may result in unstable operation.

**Amplifier Noise Figure**

In many instances an amplifier must be designed for a low noise figure. The input matching conditions for the lowest noise figure are different from those for a high power gain, so that a reduced gain is obtained from the amplifier. In addition the match for best noise may make the amplifier only conditionally stable.

The interaction between stability factors, noise factors and gain are shown using the following example:

Design a low noise amplifier for the region of 950 to 1000 MHz using a 2SC3357 low noise transistor. The specifications of that transistor are: $NF = 1.1 \text{ dB TYP.}, \quad Ga = 7.5 \text{ dB TYP. @ VCE = 10 V, IC = 7 mA, f = 1 GHz}$.

![Figure 6. Noise figure versus Collector Current (2 2SC3357 data sheet from NEC)](image)

The noise figure will vary with frequency. At $< 1 \text{MHz}$ the noise rises due to $1/f$ noise at a rate of 20 dB per decade. This $1/f$ noise is not normally included in the model for the transistor, since external components such as coupling capacitors are likely to dominate the noise figure at those frequencies. At high frequencies the noise rises because of changes in the transistor parameters. The noise figure in these the regions rises at 40 dB per decade. In addition the noise voltage changes with collector current and voltage across the transistor. The variation of noise figure for the transistor is shown in figure 6.

To start the design use a basic amplifier as shown in figure 7. The biasing resistors are chosen to cause a 7 mA collector current, corresponding to the minimum noise figure.
The Noise figure and gain can now be calculated using computer simulation and are shown in figure 8. The rise in noise figure at low frequency is due to the input coupling capacitor causing an impedance mismatch at low frequency. The rise in noise figure at high frequency is due to changes in transistor parameters and corresponds to a drop in gain. The Supply voltage, emitter resistor and bias resistor values are tuned, to obtain the best noise figure. The values shown in figure 7 correspond to the lowest noise figure consistent with a high power gain.

![Figure 7. Basic Low Noise Amplifier](image)

![Noise Figure](image)

Figure 8. Noise figure and Gain of the Basic Amplifier

Figure 9 shows a plot of K, B and Input $\mu$ and Output $\mu$ stability factors. It can be seen that in the region of 25 MHz to 300 MHz, the amplifier is only conditionally stable. The amplifier is stable with a 50 $\Omega$ source and load impedance as shown in the gain plot of figure 8. Figure 10 shows the input and output stability circles plotted at 100 MHz, confirming that the amplifier is only conditionally stable. The input stability circle is shown in pink and the output stability circle is shown in blue. It can be seen that the output stability circle causes the biggest problem. Figure 11 shows the input and output stability circles over a 50 to 1500 MHz sweep. For an unconditionally stable amplifier, the entire Smith Chart can be seen.
Figure 9. Stability factors for the amplifier.

Figure 10. Stability Input and Output Circles at 100MHz.

Figure 11. Stability Input and Output Circles over 50-1500MHz.
Figure 12 shows the constant Noise figure circles and the constant gain circles at 100 MHz. Note that the centre of the constant noise circles lie close the origin of the Smith Chart, so that the existing 50 Ω match is good for the best noise figure. The centres for the circles for available power are nowhere near the centre of the Smith Chart, so that a match for noise figure will not result in the highest power gain. Normally the first stage of a low noise amplifier is matched for the lowers noise figure and the second stage is matched for the highest power gain.

In the region of 950 to 1000 MHz the constant gain and noise circles are very different, as shown in figure 13, so that the input matching needs to be changed to obtain the best noise figure. Again, the centres of constant noise figure and those of constant available power, do not coincide, so that one must either match for the best noise figure or the best power
gain. By comparing figure 11 and figure 13, it can be seen that matching for the lowest noise at 975 MHz, will result in a worse stability, since the operating point is closer to the stability circles.

**Improving the Noise Figure**

To improve the noise figure at 975 MHz, LC tuning elements C3, L1 and L2 are added and the noise figure was tuned to give the lowest noise figure. As part of that optimisation process, the value of the coupling capacitors were reduced to prevent oscillations at low frequency. This results in the circuit shown in figure 14.

![Figure 14. Low Noise amplifier for 950-1000 MHz.](image)

![Figure 15. Noise Figure and Gain of the 950-1000 MHz amplifier.](image)

By comparing figure 8 and figure 15, it can be seen that the noise figure at 950-1000 MHz has been improved, however the operating frequency of the amplifier is very close to the
maximum useable frequency for the transistor and it is much better to use a higher frequency transistor, so that the noise figure and gain are flatter at the operating frequency. Such transistors are however likely to be more expensive.

Figure 16. Stability Factors for the 950-1000 MHz amplifier.

Figure 17. Stability Input and Output Circles of the amplifier of fig 11. over 50-1500MHz frequency range.

Comparing figures 16 and 17 with figures 9 and 11, it can be seen that the amplifier is far less stable than the basic amplifier. The amplifier is unconditionally stable in the 950 - 1000 MHz frequency region, however it is close to instability in the 30 to 300 MHz frequency region.

Figure 18 shows the gain and noise figure circles of the amplifier at 975 MHz. Note that the centre of the noise figure circles has been shifted to the origin due to the input tuning. The output tuning has little effect on the noise figure. The best noise figure that can be
obtained is 1.68 dB. The output matching is adjusted to increase the output power at 975 MHz as much as possible.

In many practical situations, particularly with power amplifiers, the transistors used will have a falling gain at 20 dB per decade over the operating range. As shown in the above example, that can lead to instability at frequencies far removed from the operating region. Any simulation for a practical design must include a stability analysis over a very wide frequency range.

It is not possible to improve the stability by say changing the output L3, of figure 14, to a tuned network and thus reducing the bandwidth of the amplifier, since that results in impedances close to the edge of the Smith Chart for some frequencies, thus worsening the stability. It is in general very difficult to ensure unconditional stability and a low noise figure. In some instances the exact source impedance, from say an Antenna, and load impedance, from following amplifiers is known and the amplifier can be kept stable. In other instances the source and load impedances are not well known, or well controlled and the amplifier design must then ensure that the amplifier is unconditionally stable.

If the amplifier is conditionally stable in the required operating frequency region, then the stability may be improved by using a branch line coupler or other 90 degree hybrid and
have two identical low noise amplifiers connected to them, as shown in figure 19. The use of 90 degree hybrids causes each of the amplifiers to see a VSWR of better than 2:1 in all instances and any reflections from the inputs of the amplifiers will end up in the load resistor connected to the port. That will improve the stability close to the operating frequency but may make the stability away from the operating frequency worse. In the circuit of figure 14, the amplifier is unconditionally stable in the 950 to 1000 MHz region, so this technique will not have any effect. The bandwidth of this amplifier is too wide for Branchline couplers to be used to improve the stability, as these are narrowband devices, however other 90 degree hybrids, like the Backward Coupled Transmission Line Hybrid can be used effectively.