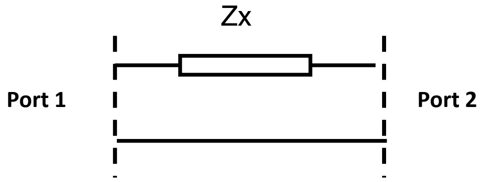
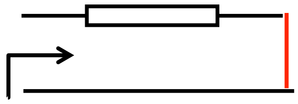
Conversion measurements of lumped elements



Reflection Measurements

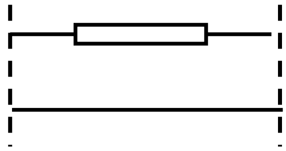


$\Gamma = S_{11}$

$$S_{11} = \frac{Z_x - Z_0}{Z_x + Z_0}$$

$$Z_x = Z_0 \frac{1 + S_{11}}{1 - S_{11}}$$

Transmission Measurements

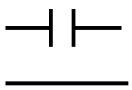


$$S_{12} = \frac{2Z_0}{Z_x + 2Z_0}$$

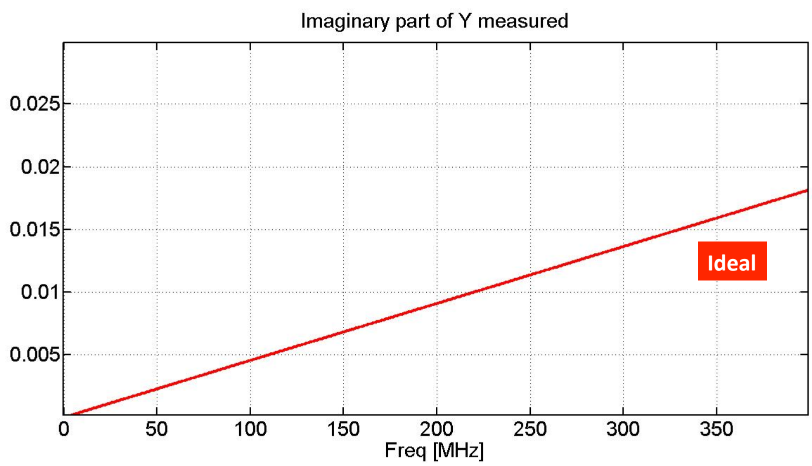
$$Z_x = Z_0 \frac{2(1 - S_{12})}{S_{12}}$$

Formulas implemented in the modern Vector Network Analyzers

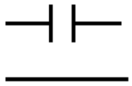
Example: capacitor



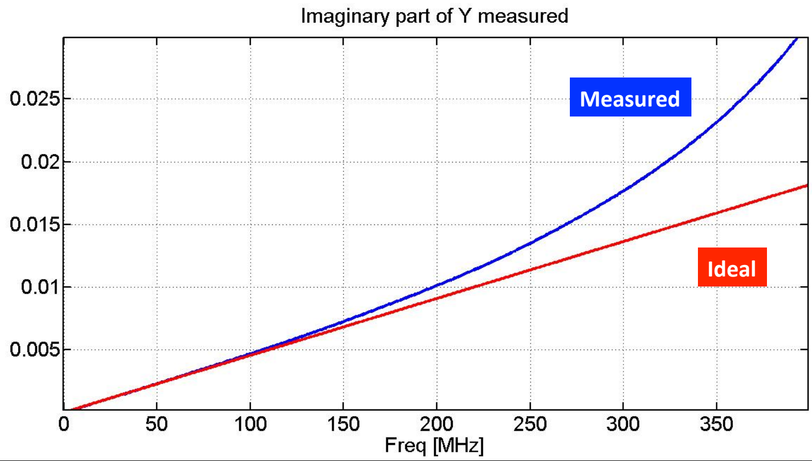
$$Y = j\omega C$$



Example: capacitor

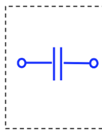


$$Y = j\omega C$$

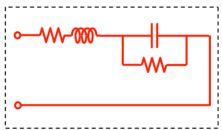


Measuring lumped elements: sources of inaccuracy

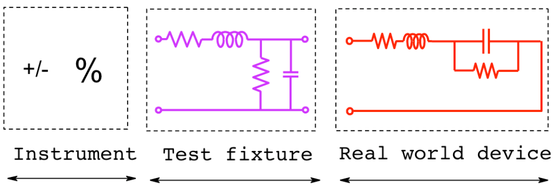
- TRUE



- EFFECTIVE



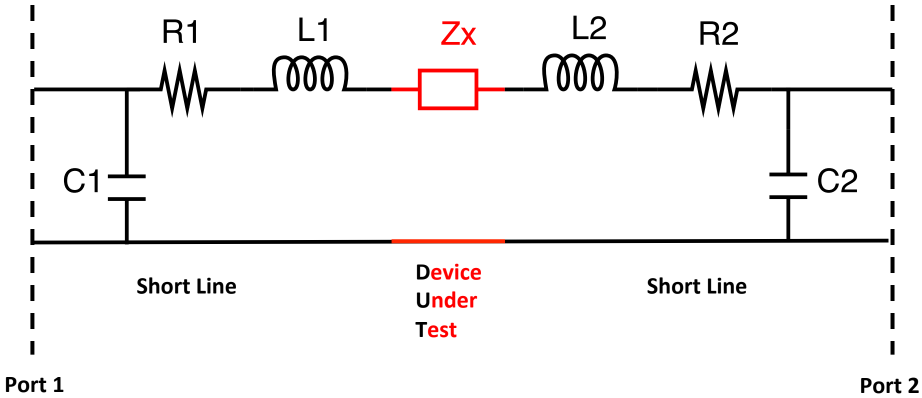
- INDICATED



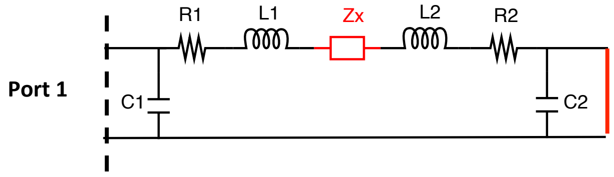
Measuring lumped elements at high frequency

Measuring lumped elements at high frequency

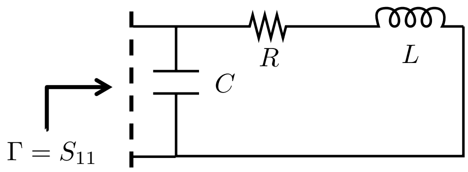
Effects of the connection to the measurement instrument



Example: Resistor at high frequency (reflection)



$$Z_x = R_x$$



$$C = C_1$$

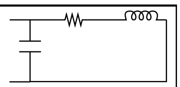
$$R = R_1 + R_2 + R_x$$

$$L = L_1 + L_2$$

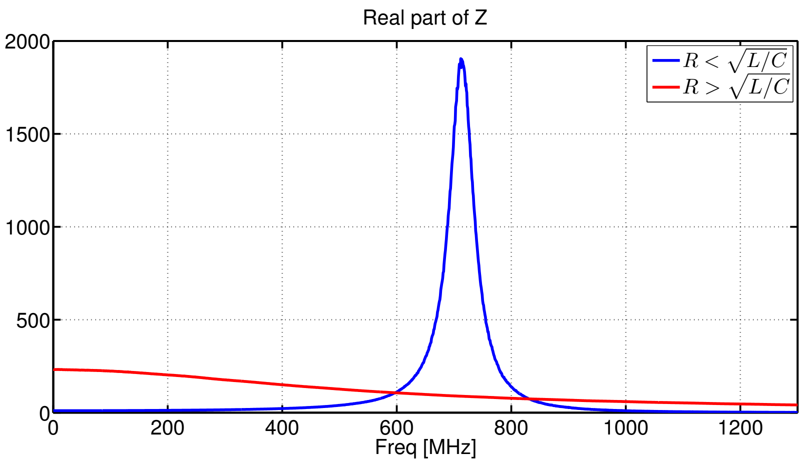
$$Y = j\omega C + \frac{1}{R + j\omega L} = \frac{R}{R^2 + (\omega L)^2} + j\omega \left[C - \frac{L}{R^2 + (\omega L)^2} \right]$$

$$Z = \frac{R}{(1 - \omega^2 LC)^2 + (\omega RC)^2} + j\omega \left[\frac{L - \omega^2 L^2 C - R^2 C}{(1 - \omega^2 LC)^2 + (\omega RC)^2} \right]$$

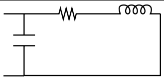
Real part of the impedance



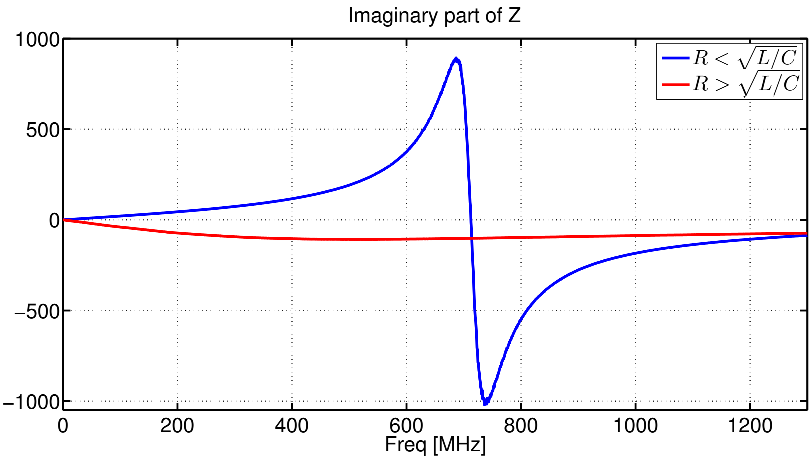
$$Z = \frac{R}{(1 - \omega^2 LC)^2 + (\omega RC)^2} + j\omega \left[\frac{L - \omega^2 L^2 C - R^2 C}{(1 - \omega^2 LC)^2 + (\omega RC)^2} \right]$$



Imaginary part of the impedance



$$Z = \frac{R}{(1-\omega^2 LC)^2 + (\omega RC)^2} + j\omega \left[\frac{L - \omega^2 L^2 C - R^2 C}{(1-\omega^2 LC)^2 + (\omega RC)^2} \right]$$

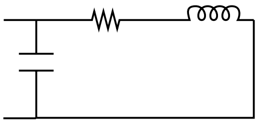


Lower frequency limit

$$Z = \frac{R}{(1-\omega^2 LC)^2 + (\omega RC)^2} + j\omega \left[\frac{L - \omega^2 L^2 C - R^2 C}{(1-\omega^2 LC)^2 + (\omega RC)^2} \right]$$

$$Y = j\omega C + \frac{1}{R + j\omega L} = \frac{R}{R^2 + (\omega L)^2} + j\omega \left[C - \frac{L}{R^2 + (\omega L)^2} \right]$$

$$Y \approx \frac{1}{R} + j\frac{\omega}{R}\left[RC - \frac{L}{R} \right]$$



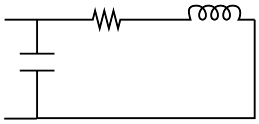
$R^2 > L/C$	**Capacitive behavior**	$Im(Y) > 0$
$R^2 < L/C$	**Inductive behavior**	$Im(Y) < 0$

Lower frequency limit (II)

$$Y \approx \frac{1}{R} + j\frac{\omega}{R}\left[RC - \frac{L}{R}\right]$$

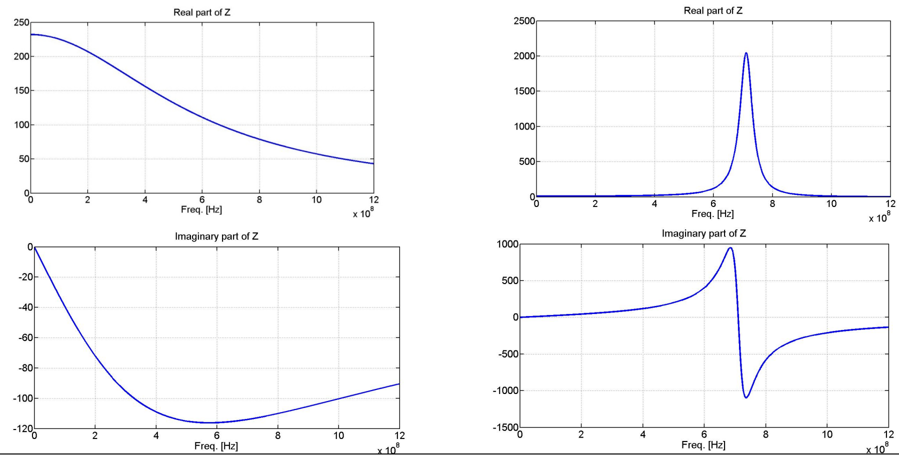
Capacitive behavior

$R > \sqrt{L/C}$



Inductive behavior

$R < \sqrt{L/C}$

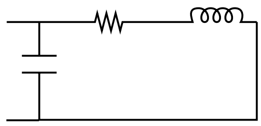


Resonance

$$Z = \frac{R}{(1-\omega^2 LC)^2 + (\omega RC)^2} + j\omega\left[\frac{L - \omega^2 L^2 C - R^2 C}{(1-\omega^2 LC)^2 + (\omega RC)^2}\right]$$

$$Y = j\omega C + \frac{1}{R + j\omega L} = \frac{R}{R^2 + (\omega L)^2} + j\omega\left[C - \frac{L}{R^2 + (\omega L)^2}\right]$$

$$Z_{IM}(\omega_0) = 0$$



$$\omega_0^2 = \frac{1}{LC}\left[1 - \frac{RC}{L/R}\right] = \frac{1}{LC}\left[1 - \frac{1}{Q^2}\right]$$

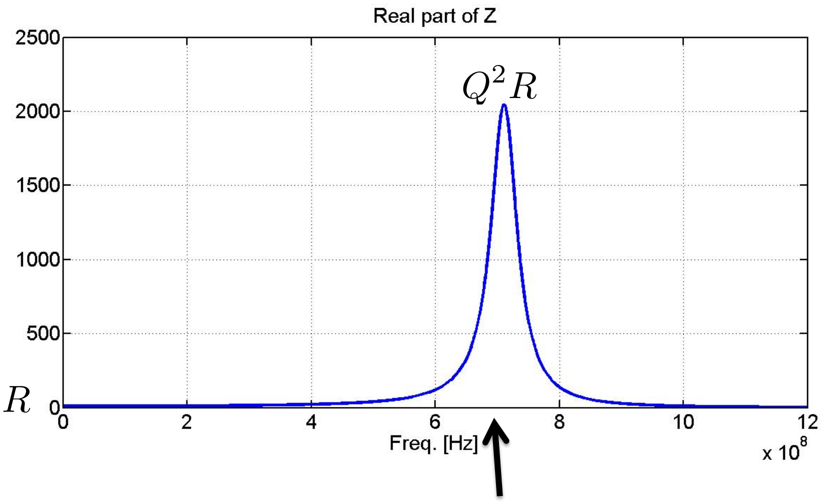
$$Q = \sqrt{L/C}/(R)$$

$$Z(\omega_0) = Q^2 R$$

$$Q = \frac{\omega_0 U_{stored}}{P_{loss}} \approx \frac{1}{\sqrt{LC}}\frac{LI^2/2}{RI^2/2} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Resonance (II)

$$Q^2 \gg 1$$



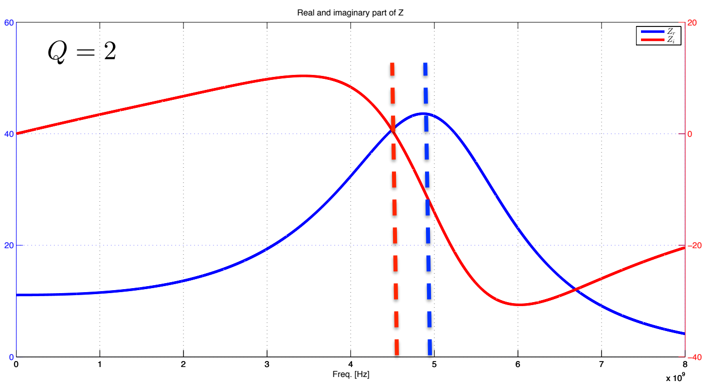
$$\approx \frac{1}{2\pi\sqrt{LC}}$$

Comment

$$Z_{IM}|_{\omega=\omega'_{01}} = 0 \quad \Rightarrow \quad \omega'_{01} = \sqrt{\frac{1}{LC}}\sqrt{1-\frac{1}{Q^2}}$$

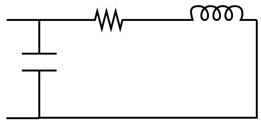
$$\frac{\partial Z_{RE}}{\partial \omega}|_{\omega=\omega'_{02}} = 0 \quad \Rightarrow \quad \omega'_{02} = \sqrt{\frac{1}{LC}}\sqrt{1-\frac{1}{2Q^2}}$$

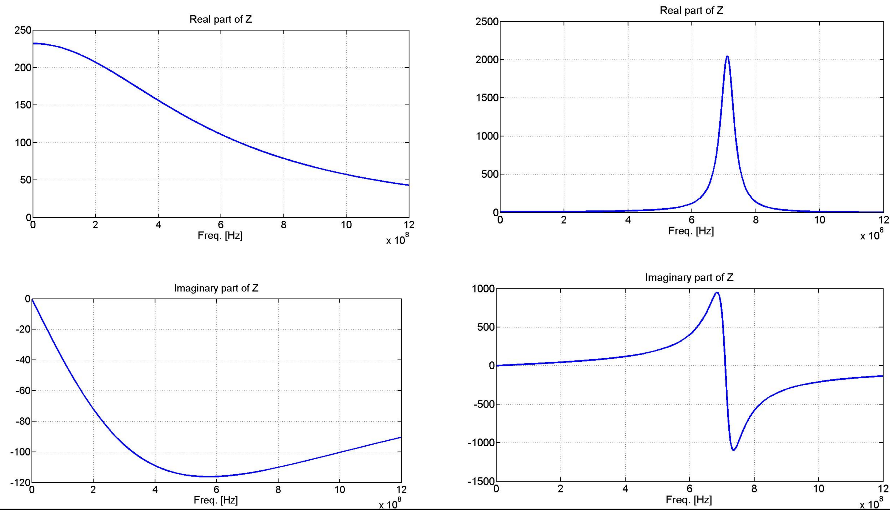
$$\omega'_{01} \neq \omega'_{02}$$



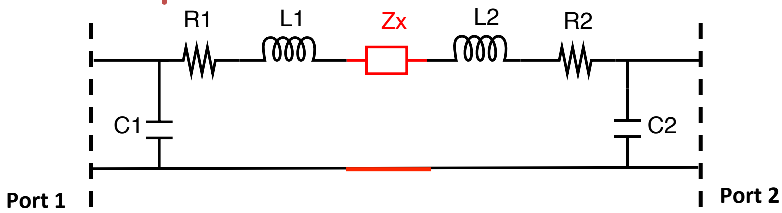
High frequency limit

$$Z = \frac{R}{(1-\omega^2 LC)^2 + (\omega RC)^2} + j\omega\left[\frac{L - \omega^2 L^2 C - R^2 C}{(1-\omega^2 LC)^2 + (\omega RC)^2}\right] \approx \frac{1}{j\omega C}$$





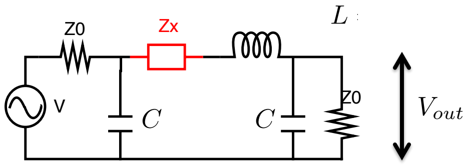
Example: transmission measurement



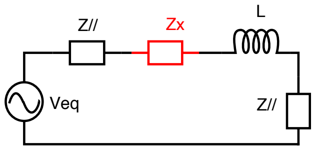
$C = C_1 = C_2$

$L = L_1 + L_2$

$R_1, R_2 \approx 0$



$$S_{21} = \frac{2V_{out}}{V}$$



$$V_{eq} = \frac{V}{1 + j\omega C Z_0} \qquad Z_{\parallel} = \frac{Z_0}{1 + j\omega C Z_0}$$

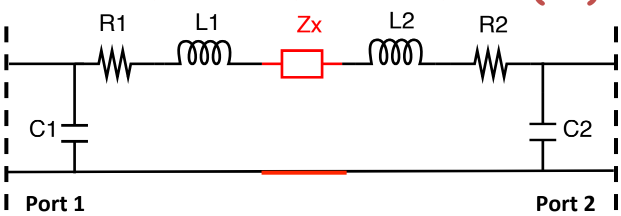
$$V_{out} = \frac{Z_{\parallel}}{Z_x + j\omega L + 2Z_{\parallel}} \frac{Z_{\parallel}}{Z_0} V$$

Example: transmission measurement (II)

$C = C_1 = C_2$

$L = L_1 + L_2$

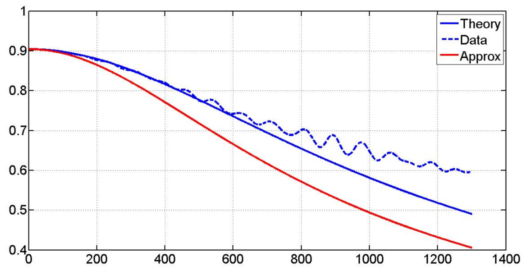
$R_1, R_2 \approx 0$



$$S_{21} = \frac{2Z_{\parallel}^2}{Z_x + j\omega L + 2Z_{\parallel}} \frac{1}{Z_0}$$

$$S_{21} \approx \frac{2Z_0}{Z_x + 2Z_0}$$

neglecting C and L (agreement @ low freq.)



Lumped element parasitic effects

Reactance vs Frequency