CAVITÀ RISONANTI

APPLICAZIONI

- GENERATORI RF DI POTENZA
- FILTRI
- ACCELERATORI / MANIPOLAZIONE DI PARTICELLE CARICHE — 100 MV/m, BANDA S
  - EFFETTI ~~[illegible]~~ PARASSITI

- CAVITÀ CILINDRICHE, MODI IN CAVITÀ
- CIRCUITO EQUIVALENTE DI MODI IN CAVITÀ
- MISURA DEI PARAMETRI DEL CIRCUITO: RIFLESSIONE
- MISURA DEI PARAMETRI DEL CIRCUITO: TRASMISSIONE

(RIFLESSIONE / TRASMISSIONE } CARATTERIZZAZIONE ESTERNA)

- ~~[illegible]~~ MISURA DEL CAMPO INTERNO
- ~~ESEMPI DI CAVITÀ MULTICELLA~~

CAVITÀ CILINDRICA: PILL BOX

MODO FONDAMENTALE

$L/a < 2{,}03 \quad TM_{010}$

$L/a > 2{,}03 \quad TE_{111}$

$$TM_{010} \begin{cases} E_z = E_0 \, J_0\left(\frac{\chi_{01}}{a}\rho\right) \\ Z_0 H_\varphi = j\, E_0 \, J_1\left(\frac{\chi_{01}}{a}\rho\right) \end{cases}$$

$\ldots \chi_{n,m} : J_n(\chi_{nm}) = 0$

$$W = 2\left[\frac{1}{4}\varepsilon_0 \int_{\tau} |E|^2 d\tau\right]$$

$$P_d = \frac{R_s}{2} \oint_S |H|^2 dS$$

$$Q = \frac{\omega_0 W}{P_d}$$

$$R_s = \sqrt{\frac{\omega_0 \mu}{2\sigma}} = \frac{1}{\sigma\delta}$$

PER TM_{010} SI DIMOSTRA CHE

$$Q = \frac{\omega_0 W}{P_d} = \frac{1{,}2025 \, Z_0}{R_s\left(1 + a/L\right)}$$

PER CASA

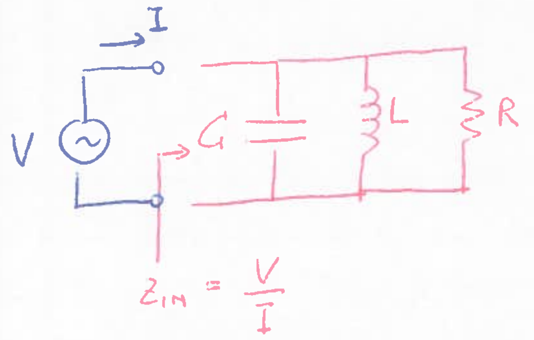
$a = 6$ cm

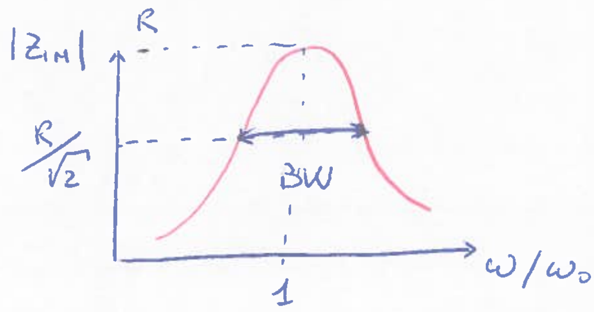
$L = 4{,}3$ cm

CALCOLARE I MODI FINO A 6 GHz

CIRCUITO EQUIVALENTE DI UN MODO IN UNA CAVITÀ

CIRCUITO SERIE E CIRCUITO PARALLELO





$$Z_{IN} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega G\right)^{-1} = \frac{R}{1 + jQ_0\delta}$$

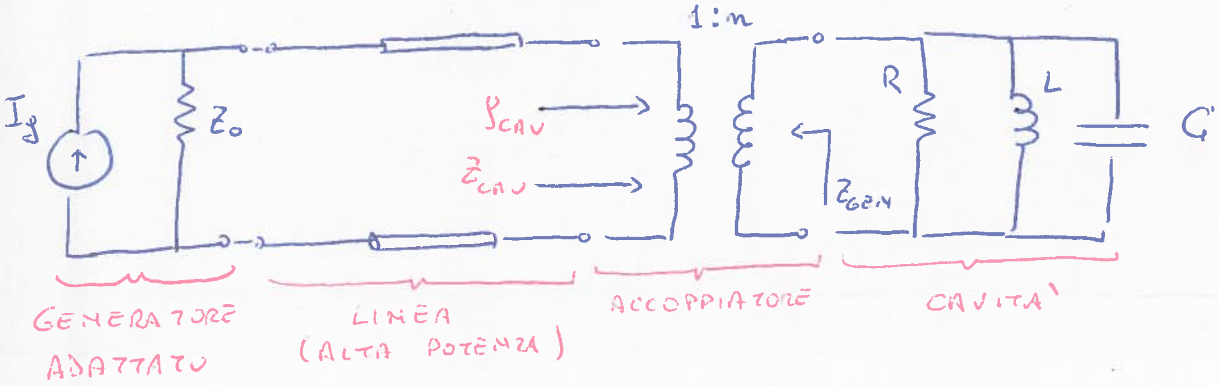
$$\delta = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \simeq 2\frac{(\omega - \omega_0)}{\omega_0} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z_{IN} \simeq \frac{R}{1 + 2jQ_0\,\Delta\omega/\omega_0} \qquad \frac{\Delta\omega_{3dB}}{\omega_0} = \frac{BW}{\omega_0} = \frac{1}{Q_0}$$

$$\text{CON} \quad Q_0 = \omega_0 \frac{W_E + W_M}{P_{LOSS}} = \frac{R}{\omega_0 L} = \omega_0 R G$$

($W_E \leftarrow G$, $W_M \leftarrow L$, $P_{LOSS} \leftarrow R$)

ACCOPPIAMENTO IN CAVITA'



$$Z_{CAV} = \frac{R/n^2}{1 + j Q_0 \delta} \qquad Q_0 = \omega_0 \frac{W}{P_{CAV}}$$

W EN. IMMAGAZINATA

P_{CAV} POTENZA PERSA SULLE PARETI DELLA CAVITA'

P_{EXT} POTENZA IRRADIATA ATTRAVERSO IL COUPLER E DISSIPATA SU Z_0 (IMPEDENZA DEL GENERATORE)

LOADED Q

$$Q_L = \frac{\omega_0 W}{P_{CAV} + P_{EXT}} \qquad \frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_E}$$

EXTERNAL Q

$$Q_E = \frac{\omega_0 W}{P_{EXT}}$$

COUPLING β

$$\beta = \frac{P_{EXT}}{P_{CAV}} = \frac{Q_0}{Q_E} = \frac{R}{n^2 Z_0}$$

$$\Rightarrow \quad Q_L = \frac{Q_0}{1+\beta}$$

MISURA DEI PARAMETRI DI UNA CAVITÀ

$$Z_{CAV} = \frac{\beta Z_0}{1 + j Q_0 \delta} \Rightarrow \rho_{CAV} = \frac{Z_{CAV} - Z_0}{Z_{CAV} + Z_0} = \frac{\beta - 1 - j Q_0 \delta}{\beta + 1 + j Q_0 \delta} \quad (S_{11})$$

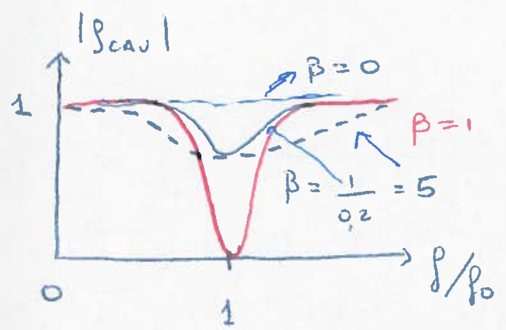
$$|\rho_{CAV}| = \sqrt{\frac{\left(\frac{\beta - 1}{\beta + 1}\right)^2 + (Q_L \delta)^2}{1 + (Q_L \delta)^2}}$$

$$|\rho_{CAV}|_{\substack{f = f_0 \\ \delta = 0}} = \frac{\beta - 1}{\beta + 1}$$

$$\angle \rho_{CAV} = -\operatorname{tg}^{-1}\left[\frac{2 \beta Q_0 \delta}{\beta^2 - 1 - (Q_0 \delta)^2}\right] \approx \frac{2\beta}{1 - \beta^2} Q_0 \delta \simeq \frac{2\beta}{1 - \beta^2} Q_0 \frac{2(f - f_0)}{f_0}$$

RETTA ....

β INFLUENZA
- → RIFLESSIONE PORTA INGRESSO @ f_0
- → P_{EXT} / P_{CAV}
- → LARGHEZZA RISONANZA

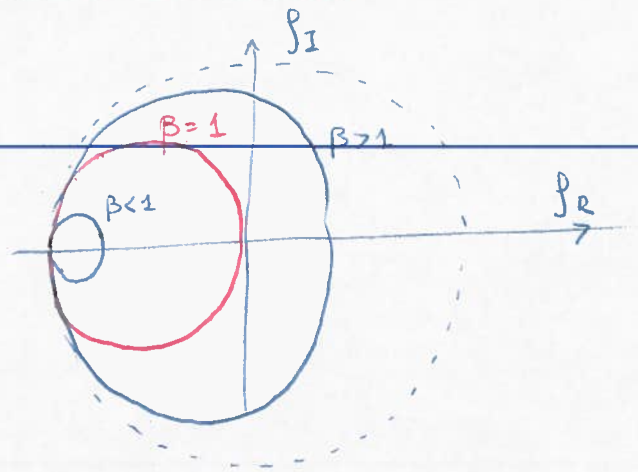


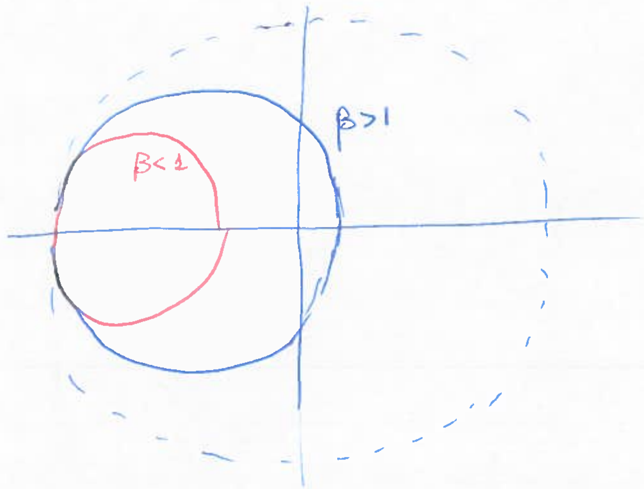
$\beta < 1$ SOTTOACCOPPIATO

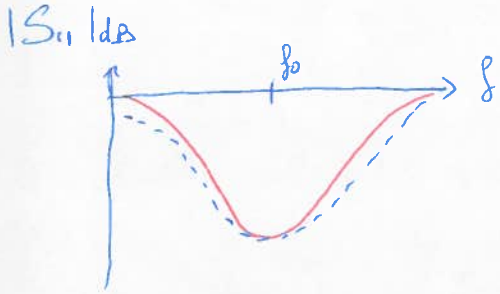
$\beta = 1$ ACCOPPIAMENTO CRITICO

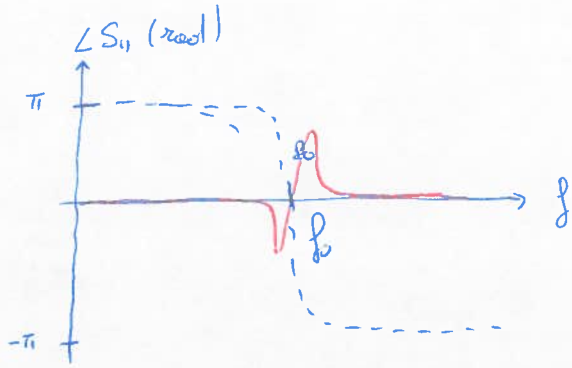
$\beta > 1$ SOVRAACCOPPIATO

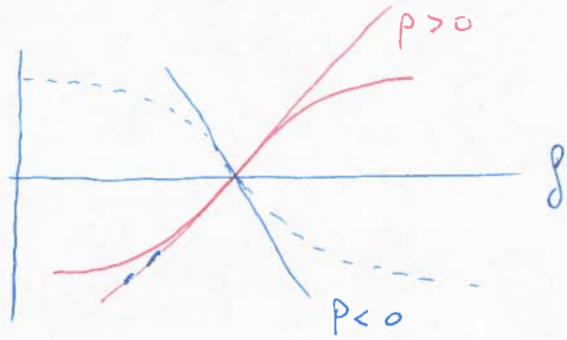
CARTA DI SMITH




β>1
β<1


|S11| dB
f0
f


∠S11 (rad)
π
-π
f0
f


p>0
p<0
f

MISURA DI β

$$|\rho_{CAV}(f_0)| = |\rho_{CAV,0}| = \left|\frac{\beta - 1}{\beta + 1}\right|$$

- $\beta > 1$

$$|\rho_{CAV,0}| = \frac{\beta - 1}{\beta + 1}$$

$$\rightarrow \beta |\rho_{CAV,0}| + |\rho_{CAV,0}| = \beta - 1$$

$$\beta = \frac{1 + |\rho_{CAV,0}|}{1 - |\rho_{CAV,0}|} = SWR(f_0)$$

- $\beta < 1$

$$|\rho_{CAV,0}| = \frac{1 - \beta}{1 + \beta} \rightsquigarrow \frac{1}{\beta} = SWR\Big|_{f = f_0}$$

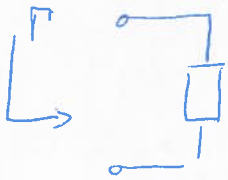
MISURA DI Q_0

- INTERPOLAZIONE $\quad |\rho_{CAV}| \longrightarrow Q_L \xrightarrow{\beta} Q_0$
- FIT LINEARE $\quad \angle \rho_{CAV} \longrightarrow \dfrac{4\beta Q_0}{(1-\beta^2) f_0} \xrightarrow{\beta} Q_0$

- CONVERSIONE $\quad \rho_{CAV} \xrightarrow{\text{CONV.}} Z_{CAV} \xrightarrow[\text{A 3dB}]{\text{BANDA}} Q_0 = \dfrac{f_0}{\Delta f_{3dB}}$

CONVERSION MEASUREMENT FOR Q_0

STARTING POINT

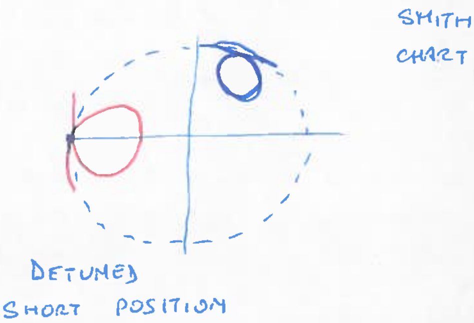


$$Z_{CAV} = \frac{\beta Z_0}{1 + j Q_0 \delta}$$

VNA MEASURES $S_{11} = \Gamma$

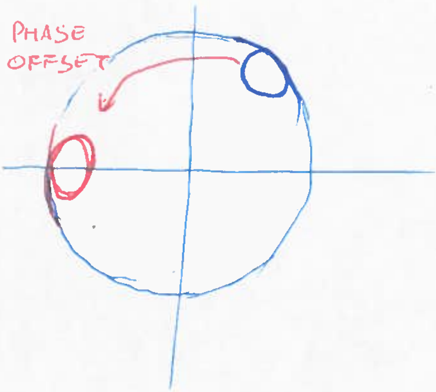
$$Z_{CAV} = Z_0 \frac{1 + \Gamma}{1 - \Gamma}$$

WHEN

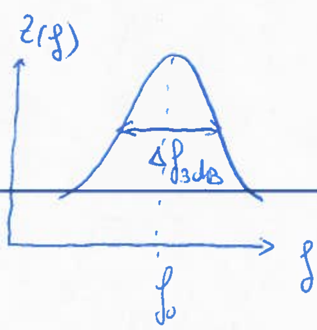


IN REALITY THE CIRCLE IS SOMEWHERE ELSE...

STEP 1 TAKE THE CIRCLE IN DETUNED SHORT POSITION WITH PHASE OFFSET



STEP 2 APPLY THE CONVERSION

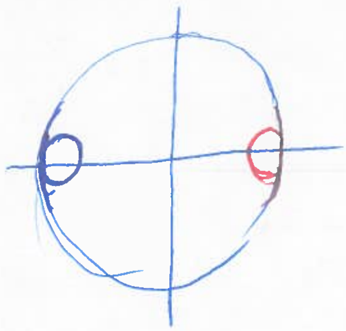


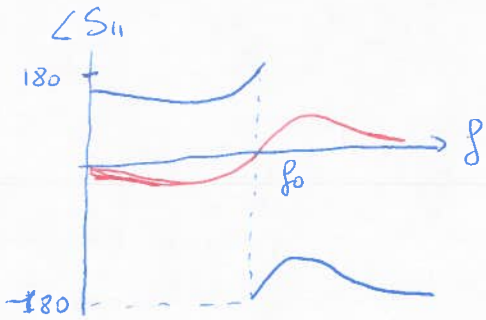
STEP 3

$$\frac{\Delta f_{3dB}}{f_0} = Q_0 \qquad \beta = \frac{Z(f_0)}{Z_0}$$

NOTE ON STEP 2

NEAR THE SHORT POSITION $\phi \approx 180^\circ$

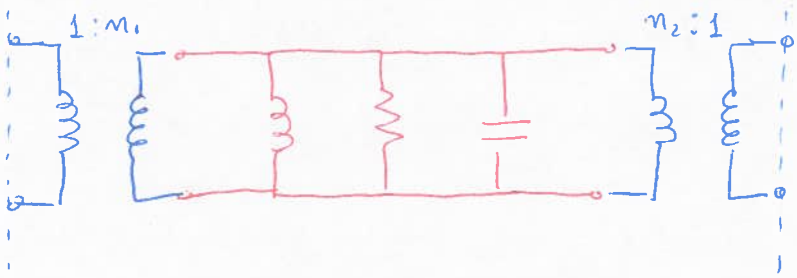


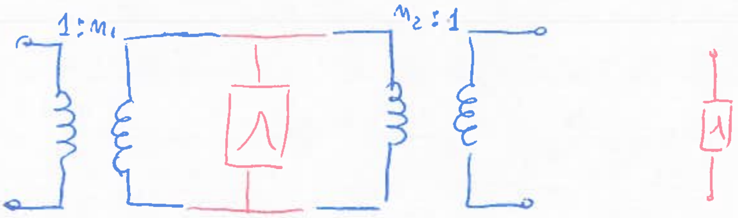


<u>HINT</u>

GO TO THE DETUNED OPEN POSITION, TAKING $\angle S_{11}(f_0) = 0$ AND THEN ADD 180° PHASE OFFSET

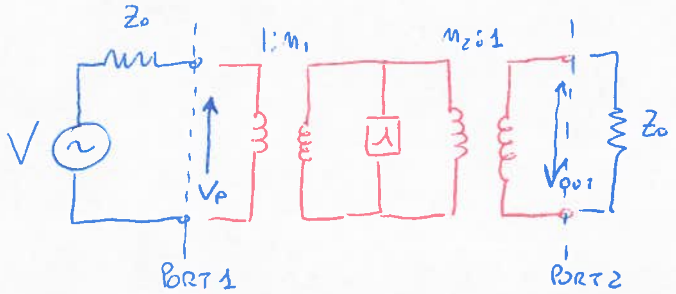
CAVITY TRANSMISSION MEASUREMENT





$$Z = \frac{R}{1 + jQ_0\delta}$$

COMPUTATION OF S_{21}



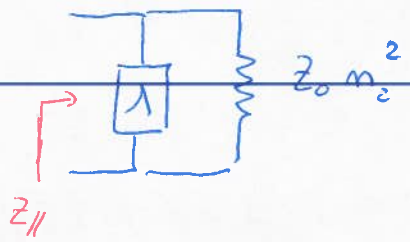
$$S_{12} = \frac{V_{OUT}}{V_{INC}}$$

$$V_p = (1 + \Gamma)\, V_{INC}$$

$$V_{OUT} = V_p \frac{n_1}{n_2}$$

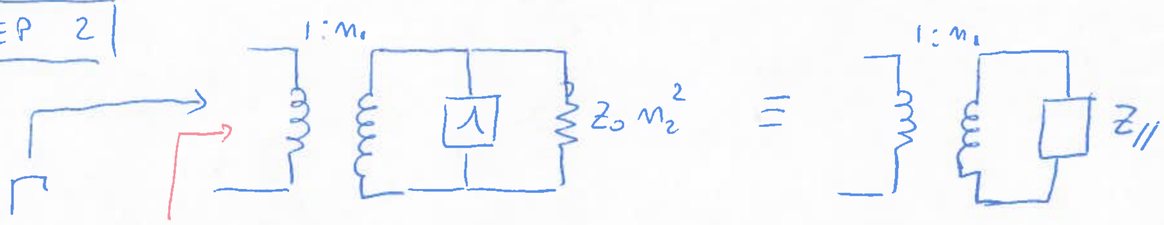
$$S_{12} = \frac{V_{OUT}}{V_{INC}} = (1 + \Gamma)\frac{n_1}{n_2}$$

STEP 1



$$Z_{\parallel} = \frac{\frac{R}{1+jQ_0\delta} \cdot Z_0 n_2^2}{\frac{R}{1+jQ_0\delta} + Z_0 n_2^2} = \frac{R}{1 + \beta_2 + jQ_0\delta} \quad \text{IF} \quad \frac{R}{n_2^2} = \beta_2 Z_0$$

STEP 2



$$Z_I = \frac{Z_{//}}{n_1^2} = \frac{R/n_1^2}{1+\beta_2+jQ_0\delta} = \frac{\beta_1 Z_0}{1+\beta_2+jQ_0\delta}$$

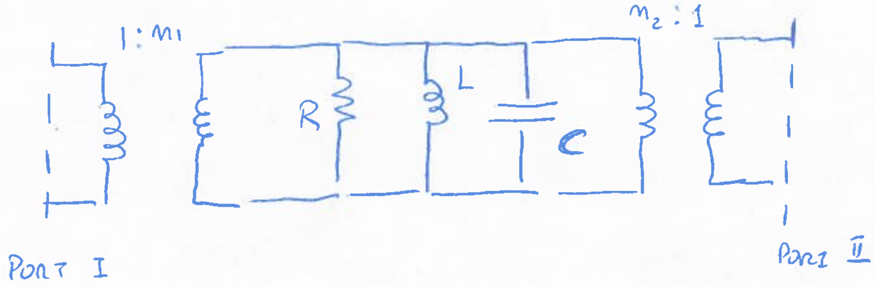
$$\Gamma = \frac{Z_I/Z_0 - 1}{Z_I/Z_0 + 1} = \frac{\beta_1 - 1 - \beta_2 - jQ_0\delta}{1+\beta_1+\beta_2+jQ_0\delta}$$

$$1+\Gamma = \frac{2\beta_1}{1+\beta_1+\beta_2+jQ_0\delta} \qquad \left(= \frac{2\, Z_I/Z_0}{Z_I/Z_0 + 1} \right)$$

$$S_{12} = (1+\Gamma)\frac{n_1}{n_2} \overset{*}{=} \qquad \frac{\beta_1 n_1}{n_2} = \frac{R/Z_0}{n_1 n_2} = \sqrt{\beta_1\beta_2}$$

$$\overset{*}{=} \frac{2\sqrt{\beta_1\beta_2}}{1+\beta_1+\beta_2+jQ_0\delta}$$

SUMMARY



$$S_{12} = \frac{2\sqrt{\beta_1 \beta_2}}{1 + \beta_1 + \beta_2 + j Q_0 \delta} \qquad \beta_J = \frac{R/Z_0}{n_J^2}$$

$$S_{11} = \frac{\beta_1 - 1 - \beta_2 - j Q_0 \delta}{1 + \beta_1 + \beta_2 + j Q_0 \delta}$$ Z_0 @ PORT II

$$\Gamma_1 = \frac{\beta_1 - 1 - j Q_0 \delta}{1 + \beta_1 + j Q_0 \delta}$$ OPEN @ PORT II

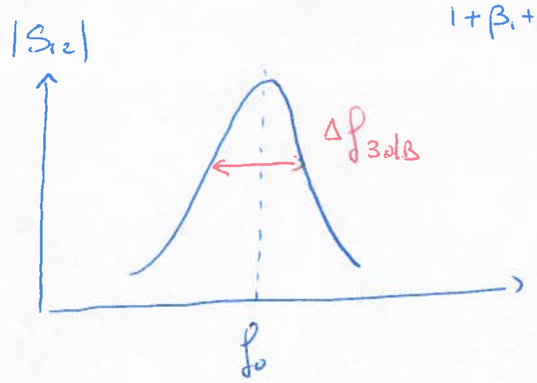
$$S_{22} = \frac{\beta_2 - 1 - \beta_1 - j Q_0 \delta}{1 + \beta_1 + \beta_2 + j Q_0 \delta}$$ Z_0 @ PORT I

$$\Gamma_2 = \frac{\beta_2 - 1 - j Q_0 \delta}{1 + \beta_2 + j Q_0 \delta}$$ OPEN @ PORT I

COMMENTS

- $S_{11}\Big|_{\text{PORT II @ } Z_0} \neq \Gamma\Big|_{\text{PORT II OPEN}}$ → TYPICAL BEHAVIOUR IN MEASUREMENTS

- $$S_{12} = \dots = \frac{2\sqrt{\beta_1\beta_2}\,/\,(1+\beta_1+\beta_2)}{1 + j\,\frac{Q_0}{1+\beta_1+\beta_2}\,\delta} = \frac{S_{12}(f_0)}{1 + j Q_L \delta}$$



$$Q_L = \frac{f_0}{\Delta f_{3dB}}$$

AUTOMATIC Q MEASUREMENT

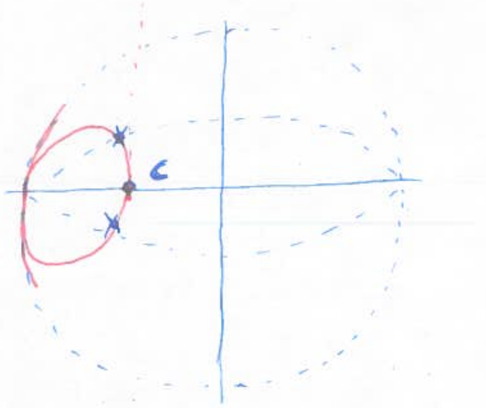
- IF $\beta_1, \beta_2 \ll 1$ $\quad Q_L \approx Q_0 \quad S_{12}(f_0) \ll 1$ BUT STILL MEASURABLE

PRACTICAL WAY TO MEASURE f_0, Q_0

- SIMMETRICAL COUPLING

$$\beta_1 = \beta_2 = \beta \qquad S_{12} = \frac{2\beta/(1+2\beta)}{1 + j Q_L \delta}$$

MISURA PARAMETRI DELLA CAVITÀ E CARTA DI SMITH



$$\frac{Z_{CAV}}{Z_0} = \frac{\beta}{1+(Q_0\delta)^2}\left[1 - j Q_0 \delta\right]$$

- PUNTI IN CUI $R_e Z = \pm I_m(Z) \Rightarrow Q_0\delta = \pm 1$

$$\delta \simeq \frac{2(f-f_0)}{f_0}$$

f_1: $\frac{2(f_1 - f_0)}{f_0} = \frac{1}{Q_0}$ (A)

$\Rightarrow \frac{f_1 - f_2}{f_0} = \frac{1}{Q_0}$

f_2: $\frac{2(f_2 - f_0)}{f_0} = -\frac{1}{Q_0}$ (B)

$\delta = 0$ $\quad Z_{CAV} = \beta Z_0$ (C) $\quad \beta = \frac{Z \text{ IN } (C)}{Z_0}$

MISURE DI Q_0 IN CAVITA' RISONANTI

MISURE RIFLESSIONE

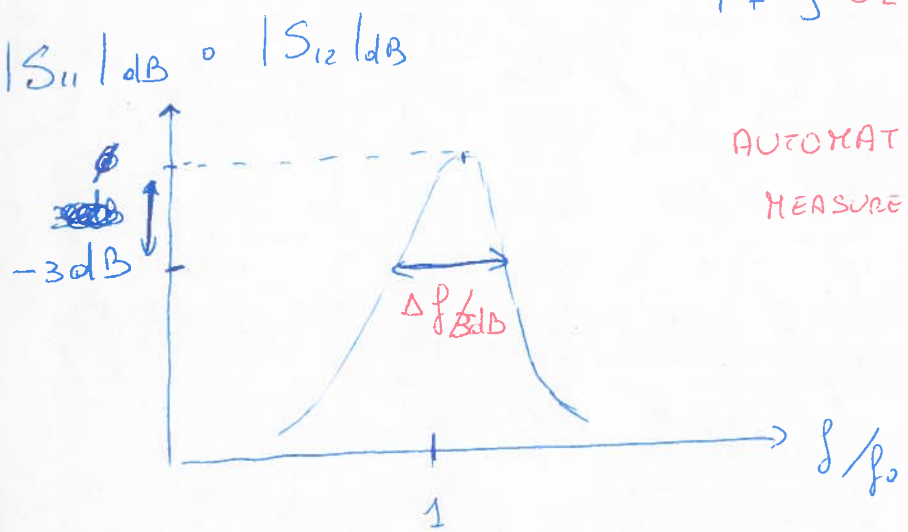
S_{11} $\xrightarrow{\text{CONV.}}$ DETUNED SHORT POSITION $\quad Z_{CAV} = \dfrac{\beta Z_0}{1 + j\, Q_0\, \delta}$

MISURA DEVE ESSERE CALIBRATA

MISURE IN TRASMISSIONE

SEMPRE $\quad S_{12} = \dfrac{S_{12}(f_0)}{1 + j\, Q_L\, \delta}$

MISURA PUO' NON ESSERE CALIBRATA



AUTOMATIC Q MEASUREMENT $\leftleftarrows$ MARKERS

$$\frac{f_0}{\Delta f_{3dB}} = \begin{cases} Q_0 & \text{PER MISURE IN RIFLESSIONE} \\ Q_L & \text{PER MISURE IN TRASMISSIONE} \end{cases}$$