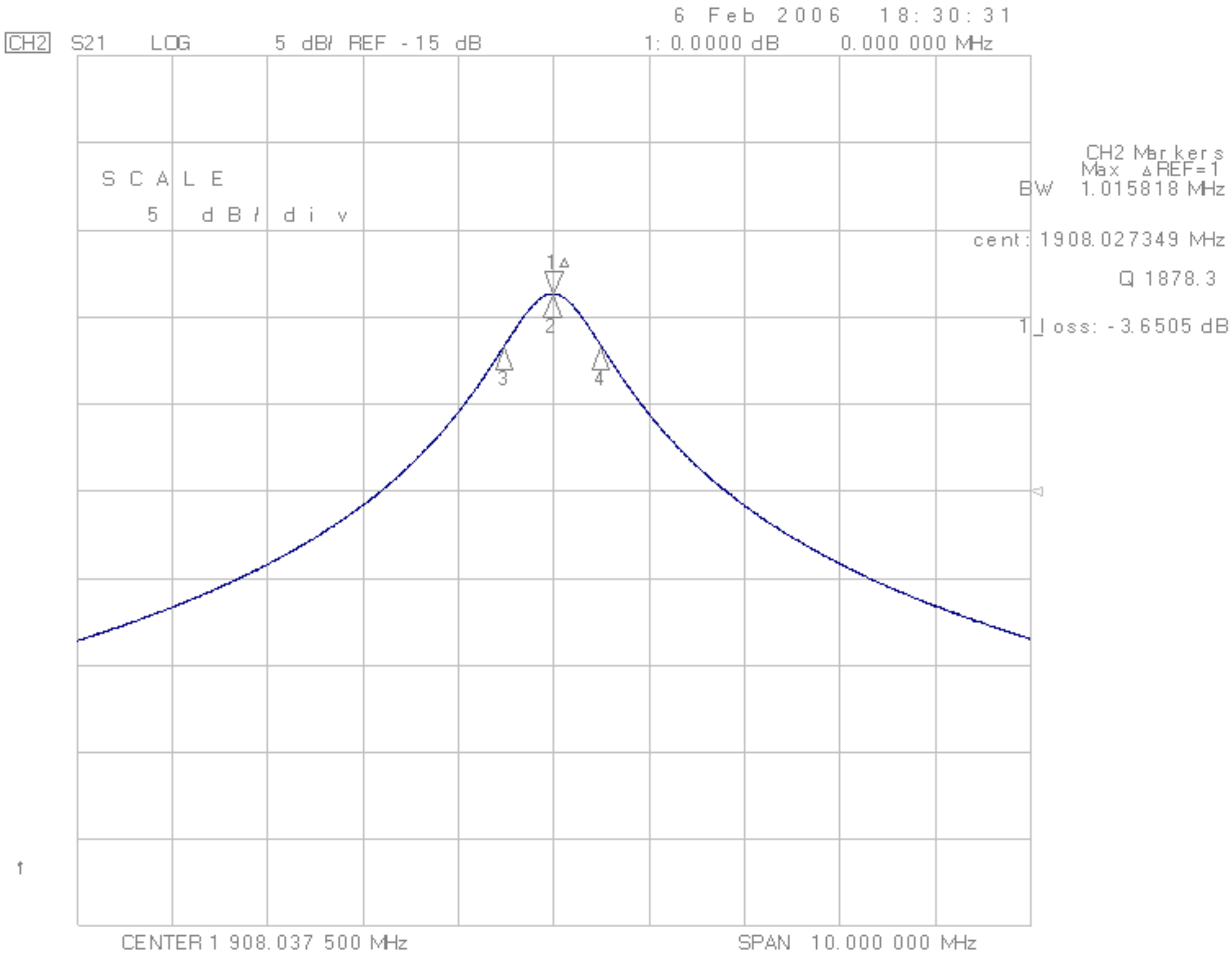
Cavity transmission measurement



Andrea.Mostacci@uniroma1.it

Slater theorem and applications

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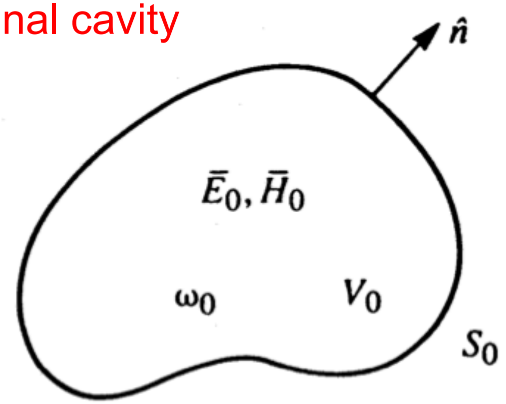
Cavity shape perturbation

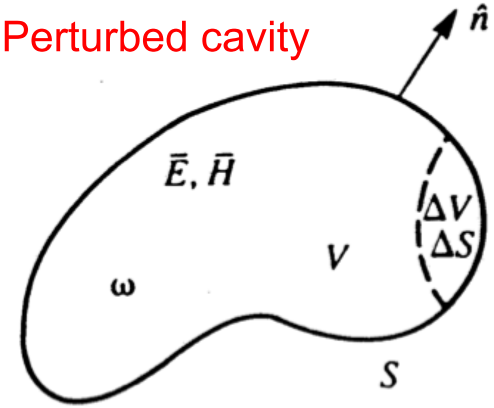
Inserting small metallic objects into a cavity or slightly deforming the shape of the cavity can be treated by the perturbation technique (**Slater theorem**)

Original cavity



$$\nabla \times \mathbf{E}_0 = -j\omega_0 \mu \mathbf{H}_0$$
$$\nabla \times \mathbf{H}_0 = j\omega_0 \varepsilon \mathbf{E}_0$$

Perturbed cavity



$$\nabla \times \mathbf{E} = -j\omega \mu \mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega \varepsilon \mathbf{E}$$

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}_0^*) - \mathbf{E}_0^* \cdot (\nabla \times \mathbf{H}) = \nabla \cdot (\mathbf{E}_0^* \times \mathbf{H}) = j\omega_0 \mu \mathbf{H}_0 \cdot \mathbf{H}_0^* - j\omega\varepsilon \mathbf{E}_0^* \cdot \mathbf{E}$$
$$\mathbf{H}_0^* \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H}_0^*) = \nabla \cdot (\mathbf{E} \times \mathbf{H}_0^*) = -j\omega\mu \mathbf{H}_0^* \cdot \mathbf{H} + j\omega_0 \varepsilon \mathbf{E} \cdot \mathbf{E}_0^*$$

$$\int_V \nabla \cdot (\mathbf{E} \times \mathbf{H}_0^* + \mathbf{E}_0^* \times \mathbf{H}) dV = \oint_S (\mathbf{E} \times \mathbf{H}_0^* + \mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S} =$$

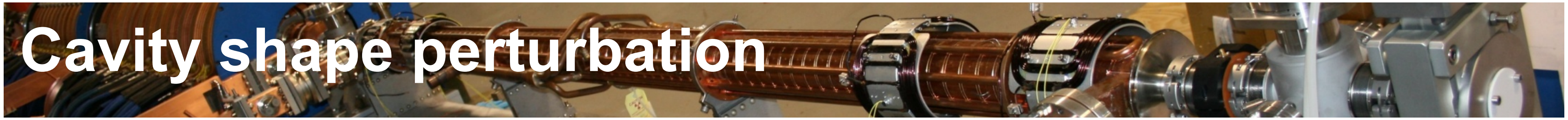
$\mathbf{n} \times \mathbf{E} = 0$ on S →

$$= \oint_S (\mathbf{E}_0^* \times \mathbf{H}) d\mathbf{S} = -j(\omega - \omega_0) \int_V (\varepsilon \mathbf{E} \cdot \mathbf{E}_0^* + \mu \mathbf{H}_0^* \cdot \mathbf{H}) dV$$

See H. Henke, RF Engineering, CAS 2000

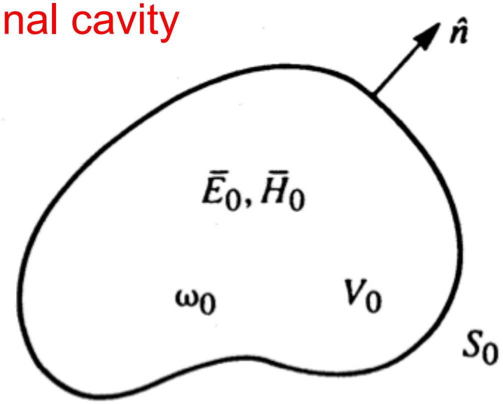
Andrea.Mostacci@uniroma1.it

Cavity shape perturbation

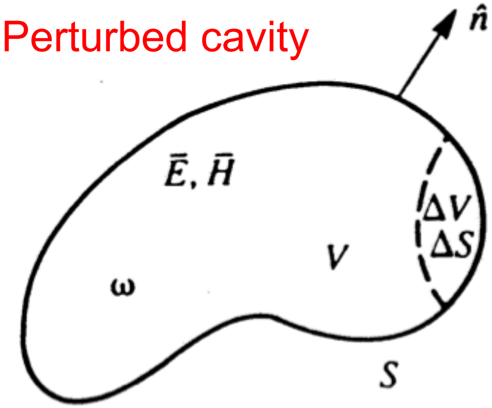


Inserting small metallic objects into a cavity or slightly deforming the shape of the cavity can be treated by the perturbation technique (**Slater theorem**)

Original cavity



Perturbed cavity



$$\int_V \nabla\cdot(\mathbf{E}\times\mathbf{H}_0^* + \mathbf{E}_0^*\times\mathbf{H})dV = \oint_S(\mathbf{E}\times\mathbf{H}_0^* + \mathbf{E}_0^*\times\mathbf{H})\cdot d\mathbf{S} =$$

$$= \oint_S(\mathbf{E}_0^*\times\mathbf{H})\cdot d\mathbf{S} = -j(\omega-\omega_0)\int_V(\varepsilon\mathbf{E}\cdot\mathbf{E}_0^* + \mu\mathbf{H}_0^*\cdot\mathbf{H})dV$$

$S = S_0 - \Delta S$

$$\oint_{S_0}(\mathbf{E}_0^*\times\mathbf{H})d\mathbf{S} - \oint_{\Delta S}(\mathbf{E}_0^*\times\mathbf{H})d\mathbf{S} = \oint_{\Delta S}(\mathbf{E}_0^*\times\mathbf{H})d\mathbf{S}$$

↑

$\mathbf{n}\times\mathbf{E}_0 = 0$ on S_0

$$\omega-\omega_0 = -j\frac{\oint_{\Delta S}(\mathbf{E}_0^*\times\mathbf{H})\cdot d\mathbf{S}}{\int_V(\varepsilon\mathbf{E}\cdot\mathbf{E}_0^* + \mu\mathbf{H}_0^*\cdot\mathbf{H})dV}$$

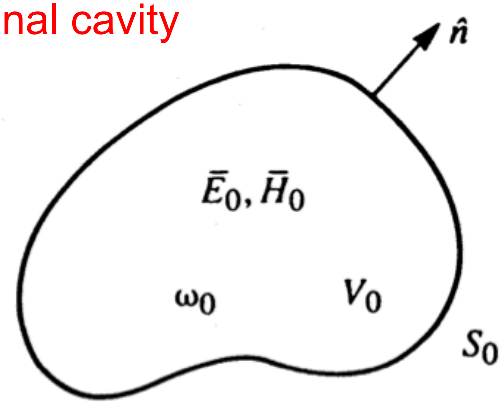
Exact (but not very useful) expression since E, H in the perturbed cavity are unknown.

See H. Henke, RF Engineering, CAS 2000

Andrea.Mostacci@uniroma1.it

Cavity shape perturbation

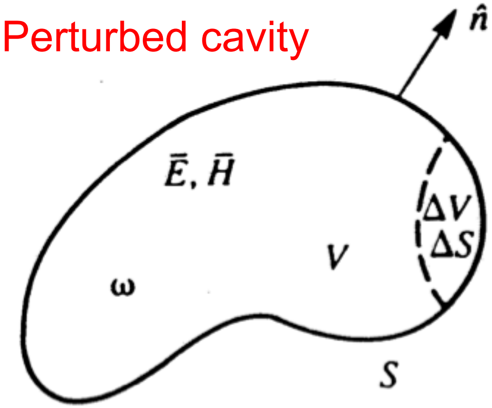
Original cavity



$$\mathbf{E} \approx \mathbf{E}_0$$
$$\mathbf{H} \approx \mathbf{H}_0$$

small perturbation

Perturbed cavity



Poyting Theor.

$$\oint_{\Delta S} (\mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S} \approx \oint_{\Delta S} (\mathbf{E}_0^* \times \mathbf{H}_0) \cdot d\mathbf{S} = -j\omega_0 \int_{\Delta V} (\varepsilon |\mathbf{E}_0|^2 - \mu |\mathbf{H}_0|^2) dV$$

$$\omega - \omega_0 = -j \frac{\oint_{\Delta S} (\mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S}}{\int_V (\varepsilon \mathbf{E} \cdot \mathbf{E}_0^* + \mu \mathbf{H}_0^* \cdot \mathbf{H}) dV}$$

It is essentially the energy stored in the cavity and it will not change much with the perturbation.

Change in stored electric/ magnetic energy

$$\frac{\omega - \omega_0}{\omega_0} \approx \frac{\int_{\Delta V} (\mu |\mathbf{H}_0|^2 - \varepsilon |\mathbf{E}_0|^2) dV}{\int_{V_0} (\varepsilon |\mathbf{E}_0|^2 + \mu |\mathbf{H}_0|^2) dV} = \frac{\Delta W_m - \Delta W_e}{W_m + W_e}$$

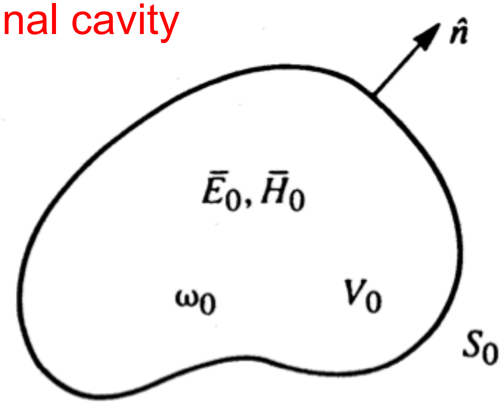
Total energy stored

See H. Henke, RF Engineering, CAS 2000

Andrea.Mostacci@uniroma1.it

Applications: tuning of a cavity

Original cavity

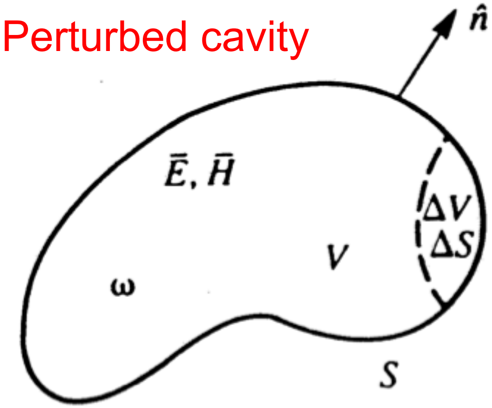


$$\mathbf{E} \approx \mathbf{E}_0$$

$$\mathbf{H} \approx \mathbf{H}_0$$

small perturbation

Perturbed cavity



$$\frac{\Delta\omega}{\omega_0} \approx \frac{\int_{\Delta V}(\mu|\mathbf{H}_0|^2 - \varepsilon|\mathbf{E}_0|^2)dV}{\int_{V_0}(\varepsilon|\mathbf{E}_0|^2 + \mu|\mathbf{H}_0|^2)dV} = \frac{1}{W_{tot}}\int_{\Delta V}\mu|\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}}\int_{\Delta V}\varepsilon|\mathbf{E}_0|^2 dV$$

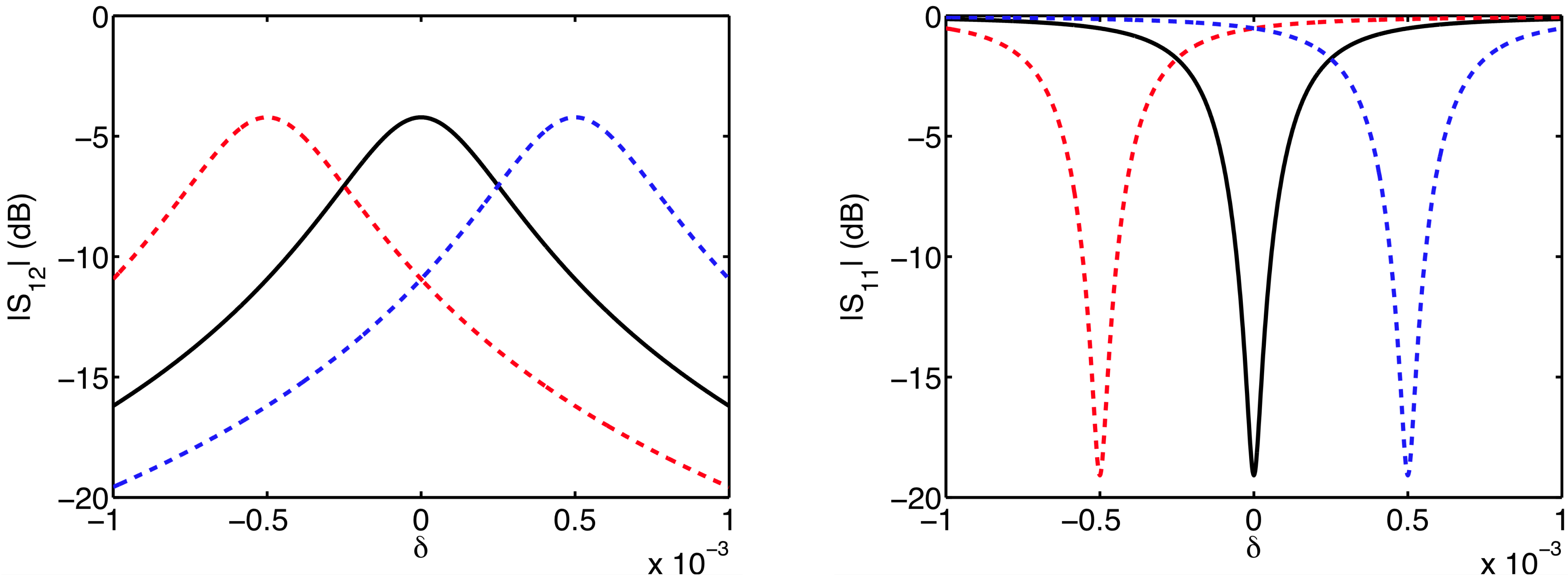
The frequency shift depends on the kind and the amplitude of the unperturbed cavity field.

Andrea.Mostacci@uniroma1.it

Applications: tuning of a cavity (S parameters)

$$\frac{\Delta\omega}{\omega_0} \approx \frac{1}{W_{tot}} \int_{\Delta V} \mu |\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}} \int_{\Delta V} \varepsilon |\mathbf{E}_0|^2 dV$$

Unperturbed **Electric Field** **Magnetic Field**

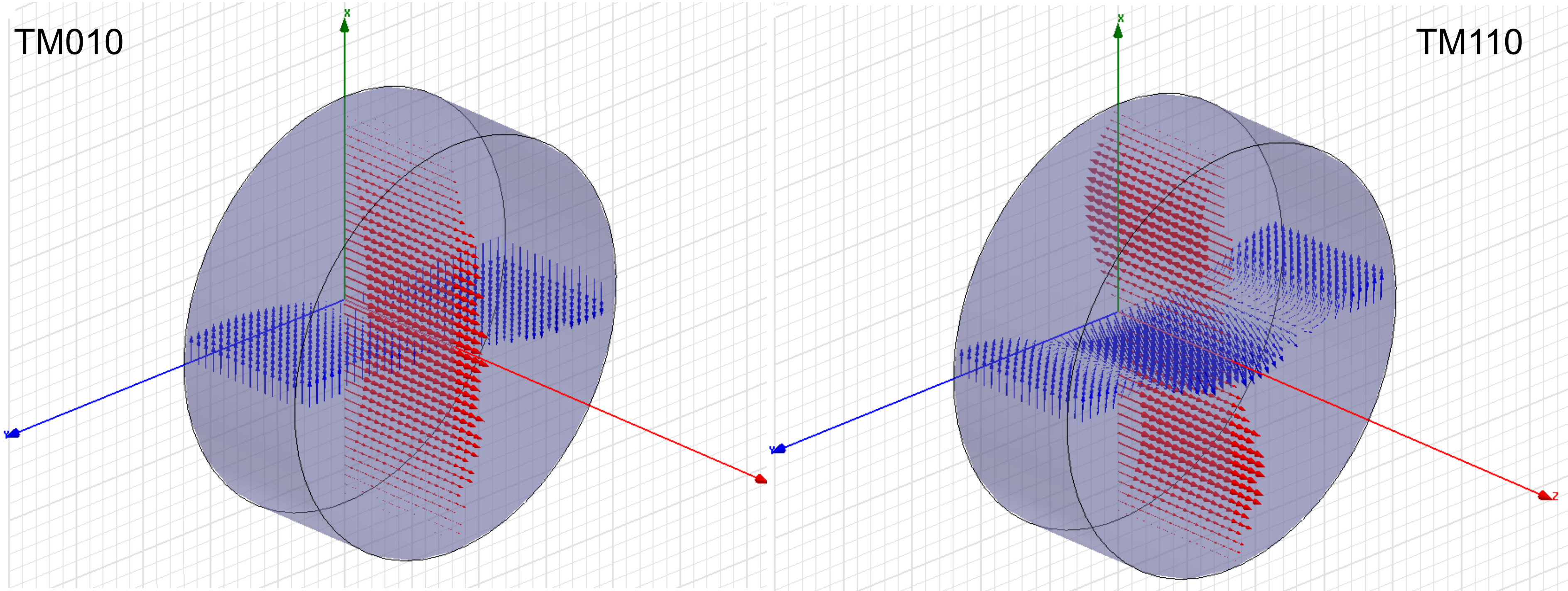


Andrea.Mostacci@uniroma1.it

Applications: tuning of different cavity modes

The same tuners affect different modes in different ways …

$$\frac{\Delta\omega}{\omega_0} \approx \frac{1}{W_{tot}} \int_{\Delta V} \mu |\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}} \int_{\Delta V} \varepsilon |\mathbf{E}_0|^2 dV$$



Task: for a given cavity with two tuners, you can tune the resonant frequency two modes simultaneously.

Courtesy of L. Ficcadenti

Applications: bead pull measurement

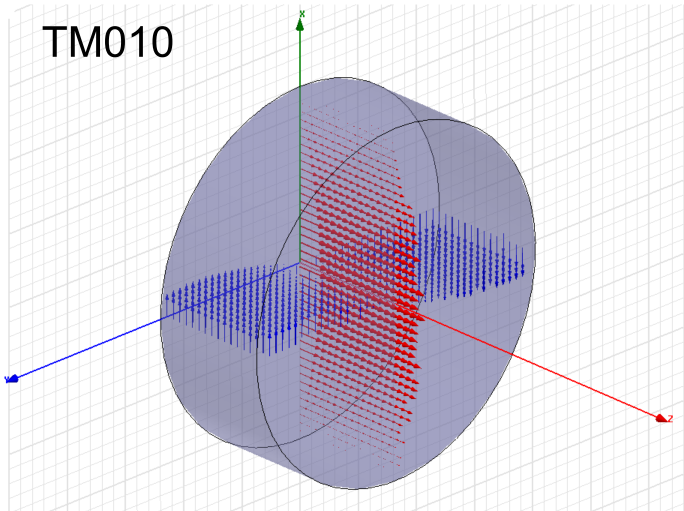
Introducing a small metallic object into the interior of the cavity should perturb the frequency in a similar way by an amount depending upon the local fields, and thus we could use the **frequency shift to measure the field strength** at an interior point.

$$\frac{\Delta\omega}{\omega_0} \approx \frac{1}{W_{tot}} \int_{\Delta V} \mu |\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}} \int_{\Delta V} \varepsilon |\mathbf{E}_0|^2 dV$$

Measurements

$$\frac{\Delta\omega}{\omega_0} \approx \left(k_{\parallel}^{H} \mu \frac{|H_z|^2}{W_{tot}} + k_{\perp}^{H} \mu \frac{|\mathbf{H}_{\perp}|^2}{W_{tot}} \right) - \left(k_{\parallel}^{E} \varepsilon \frac{|E_z|^2}{W_{tot}} + k_{\perp}^{E} \varepsilon \frac{|\mathbf{E}_{\perp}|^2}{W_{tot}} \right)$$

Theory and/or calibration in known cavities



Accelerating field on the cavity axis

$$\frac{|E_z|^2}{W_{tot}} \approx -\frac{1}{k_{\perp}^{E} \varepsilon} \frac{\Delta\omega}{\omega_0}$$

Measurements

Andrea.Mostacci@uniroma1.it

Applications: bead pull measurement

Introducing a small metallic object into the interior of the cavity should perturb the frequency in a similar way by an amount depending upon the local fields, and thus we could use the **frequency shift to measure the field strength** at an interior point.

$$\frac{\Delta\omega}{\omega_0} \approx \frac{1}{W_{tot}} \int_{\Delta V} \mu |\mathbf{H}_0|^2 dV - \frac{1}{W_{tot}} \int_{\Delta V} \varepsilon |\mathbf{E}_0|^2 dV$$

Measurements

$$\frac{\Delta\omega}{\omega_0} \approx \left(k_{\parallel}^{H} \mu \frac{|H_z|^2}{W_{tot}} + k_{\perp}^{H} \mu \frac{|\mathbf{H}_{\perp}|^2}{W_{tot}} \right) - \left(k_{\parallel}^{E} \varepsilon \frac{|E_z|^2}{W_{tot}} + k_{\perp}^{E} \varepsilon \frac{|\mathbf{E}_{\perp}|^2}{W_{tot}} \right)$$

Theory and/or calibration in known cavities

In general

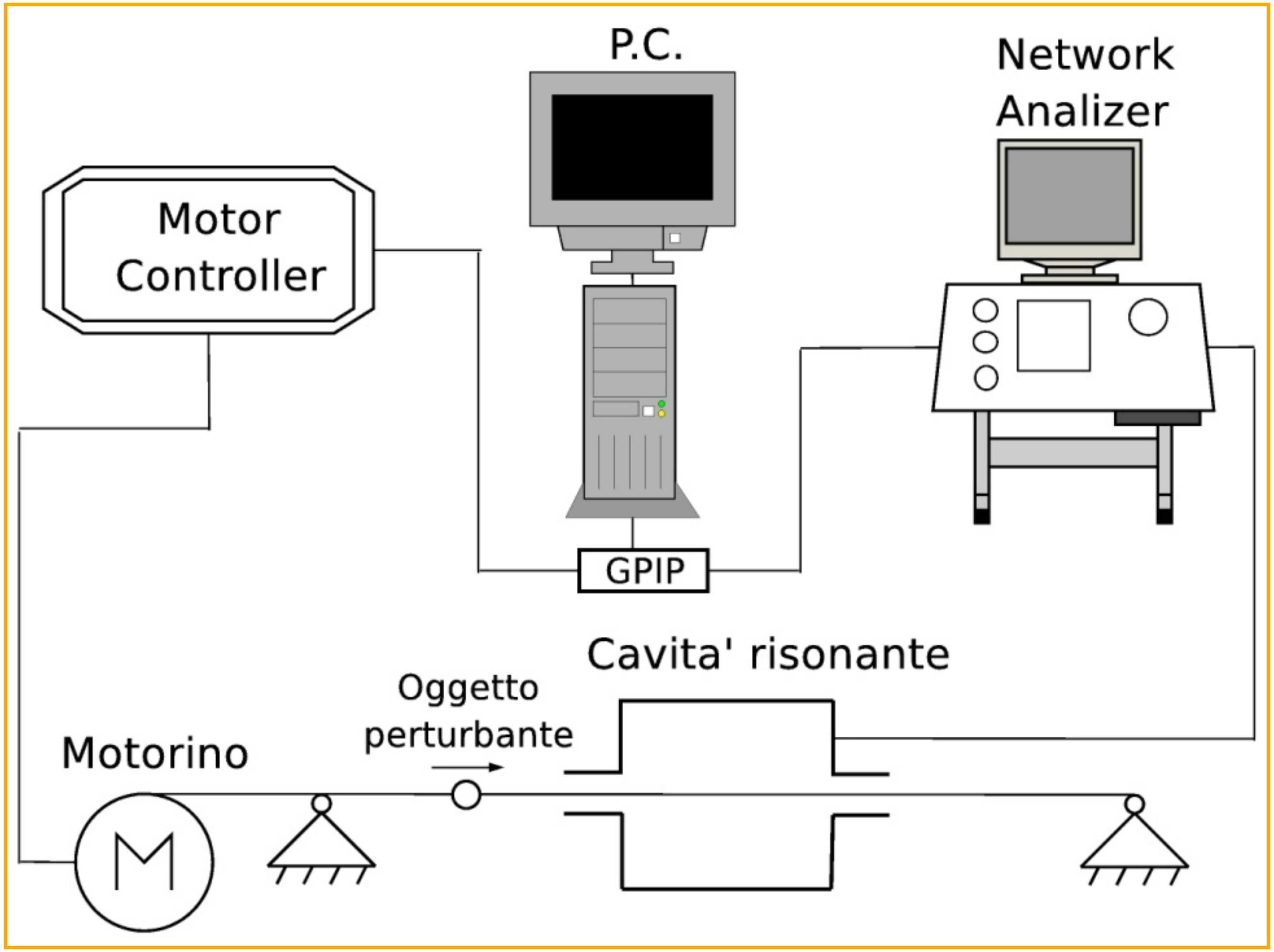
Metal objects affect E and H field

Dielectric objects affect E field

→ It is possible to measure H field with two measurements

Andrea.Mostacci@uniroma1.it

Automatic field measurement (bead pull)



Courtesy of L. Ficcadenti

Examples

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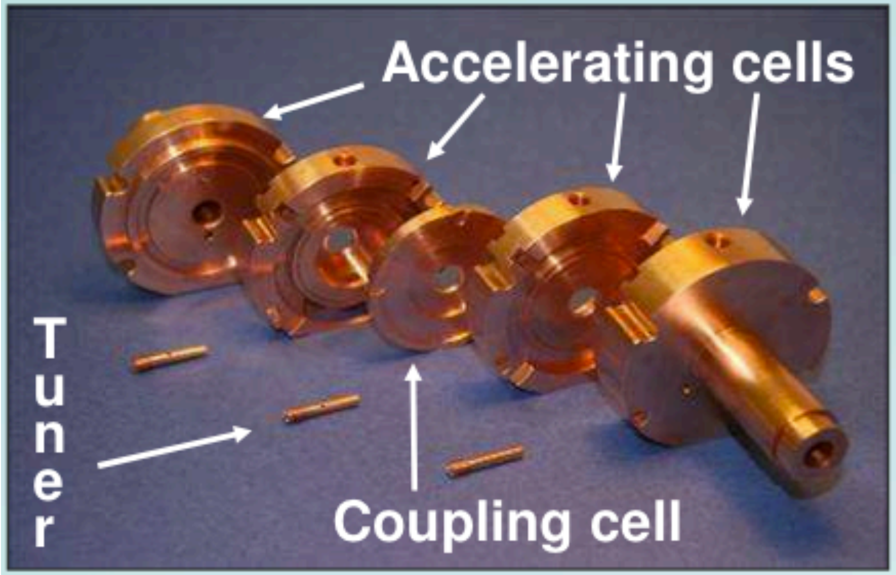
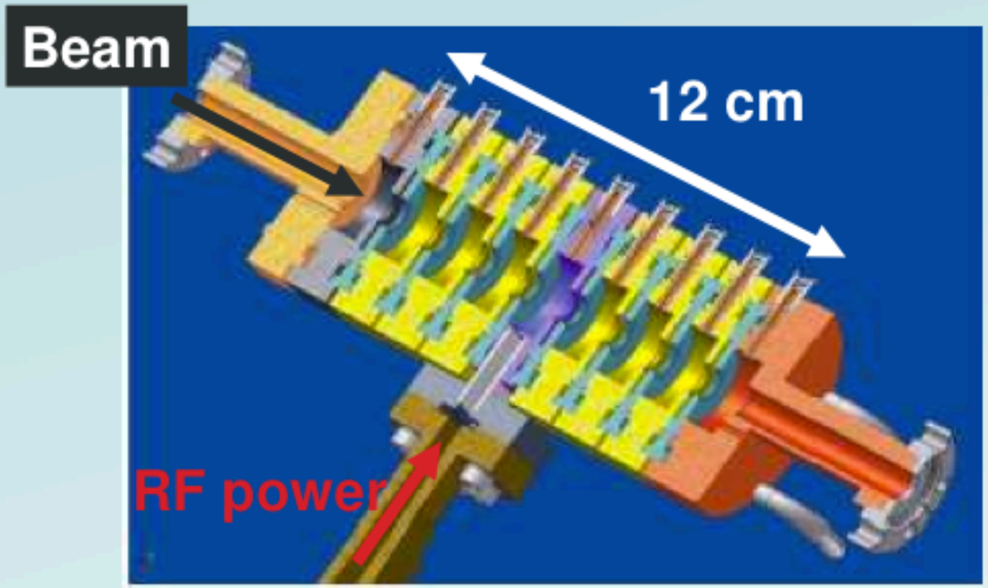
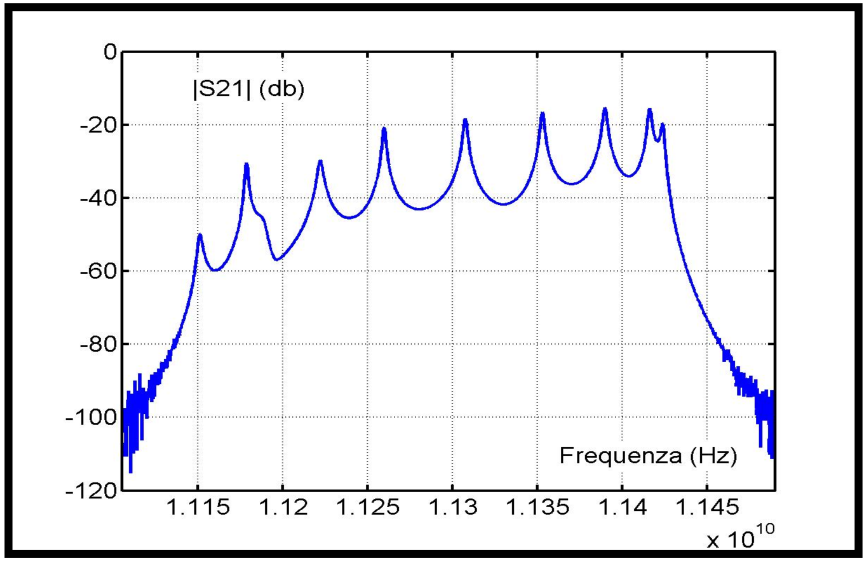
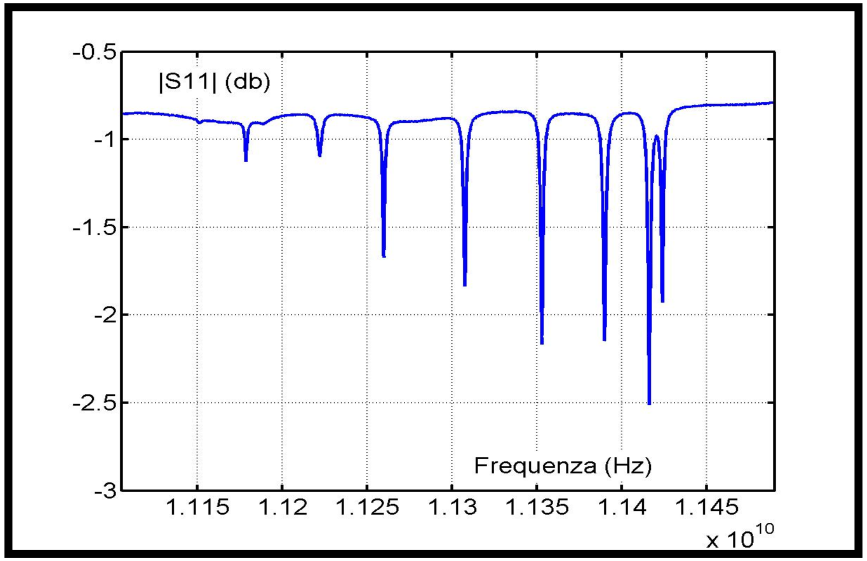
Periodic cavity: S parameters



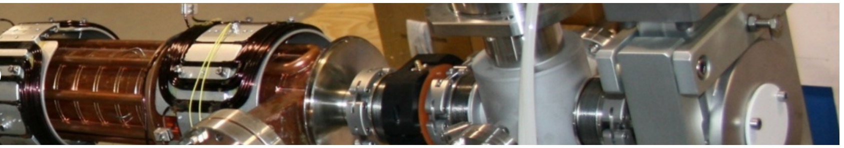


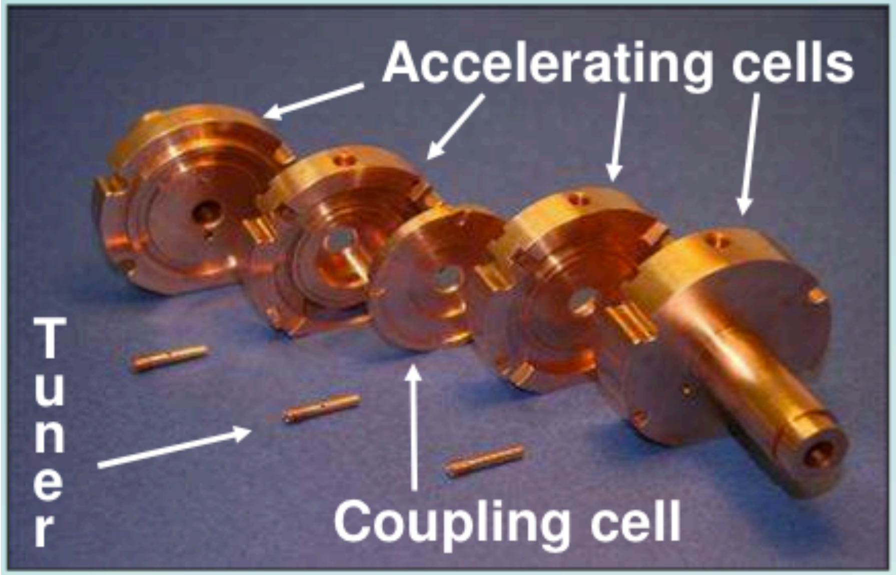


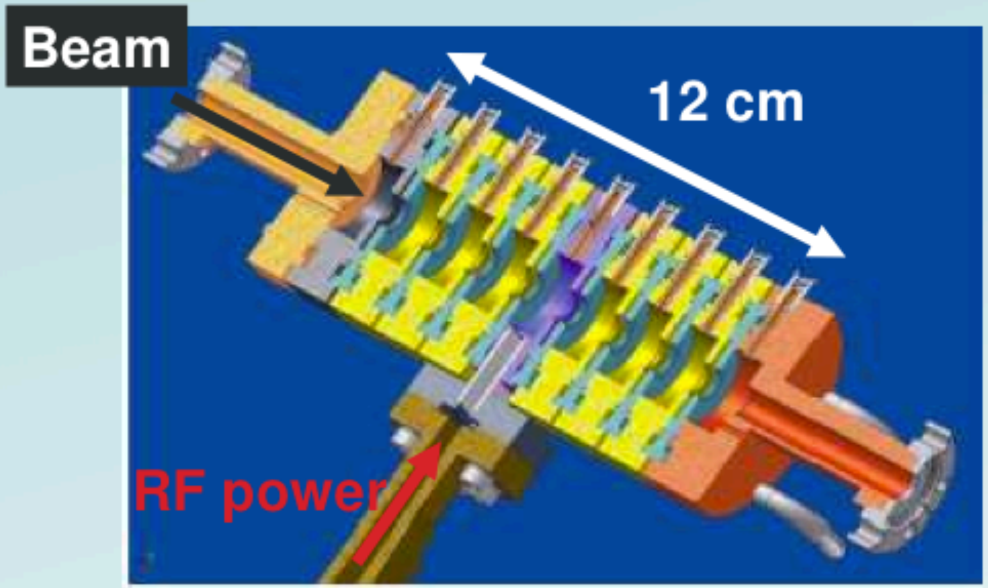


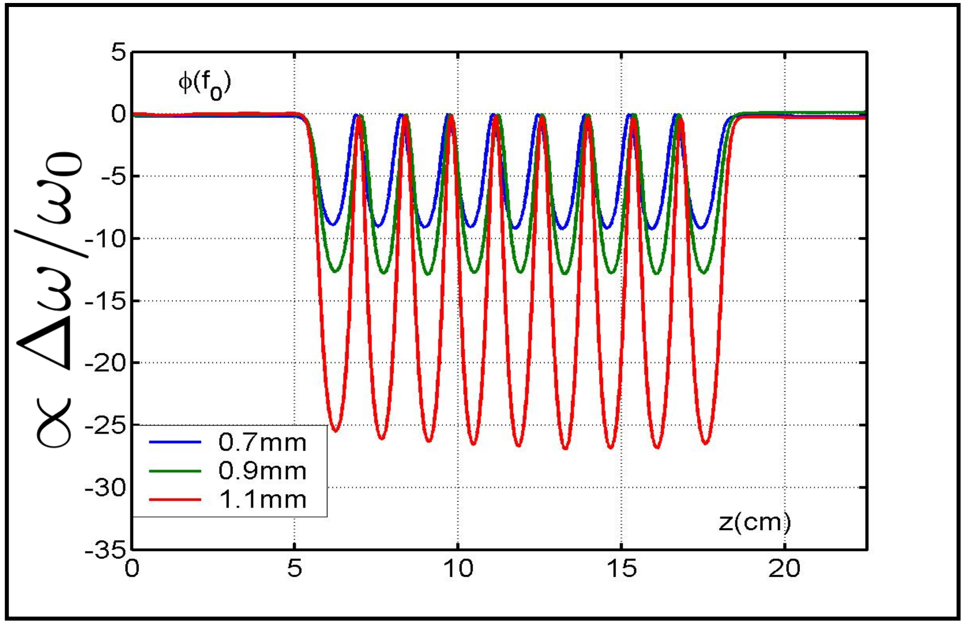
Courtesy of L. Ficcadenti

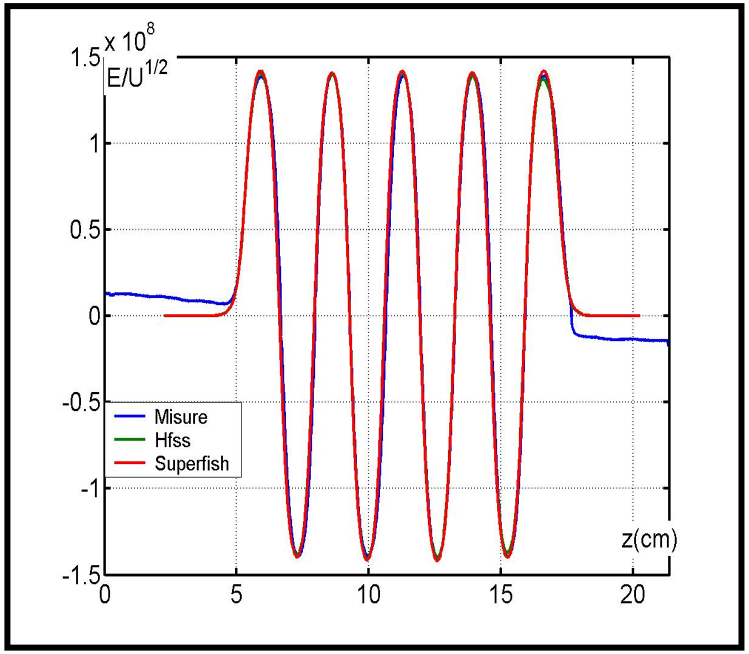
Periodic cavities: field on axis





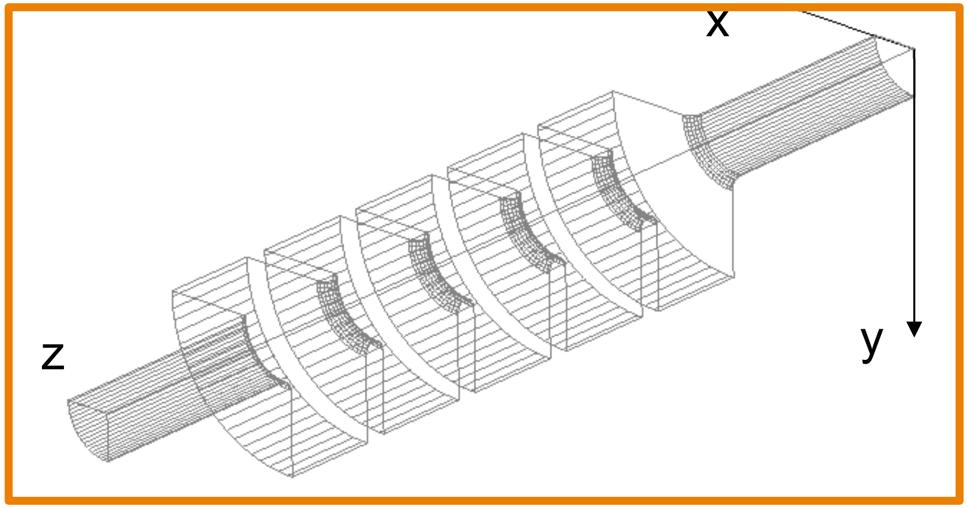






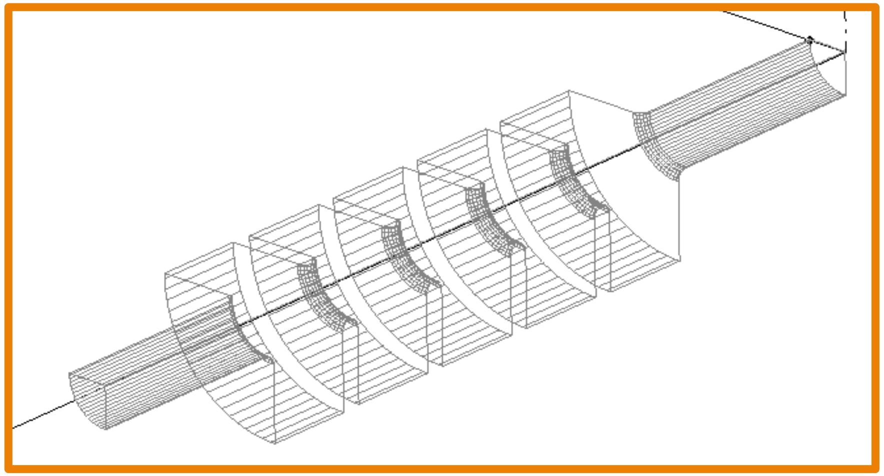
Andrea.Mostacci@uniroma1.it

RF deflector for SPARC beam



- Campo E sull'asse
- Modo π (TM_{110}-like)

- Campo H sull'asse
- Modo π (TM_{110}-like)



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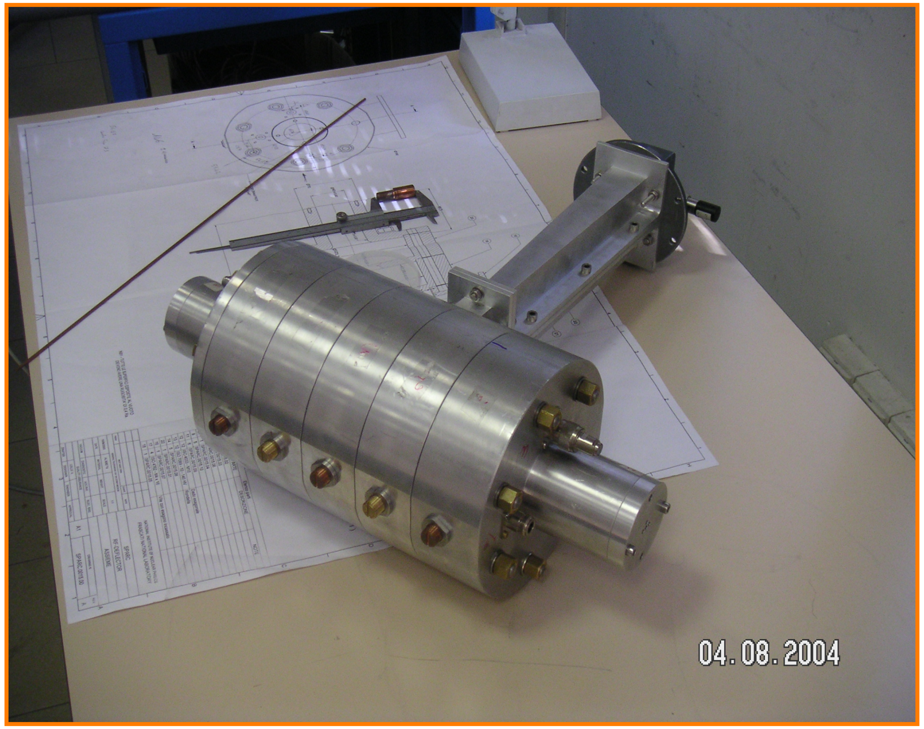
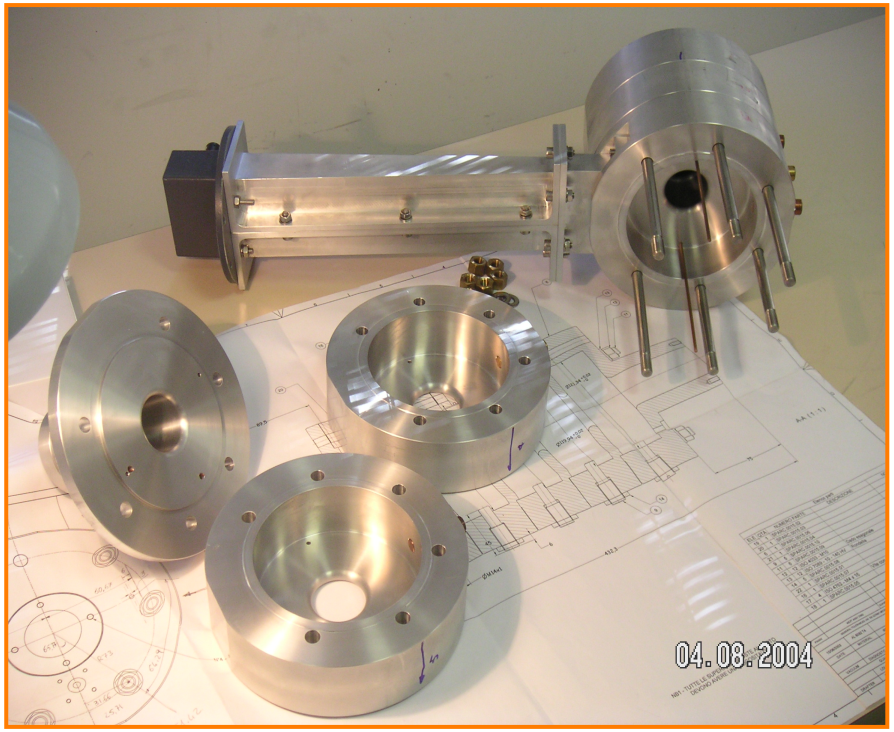
Andrea.Mostacci@uniroma1.it

RF deflector Al prototype





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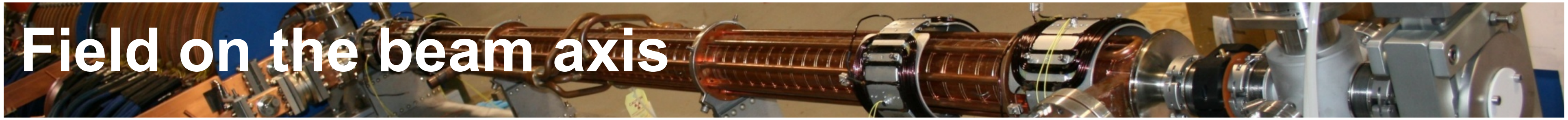
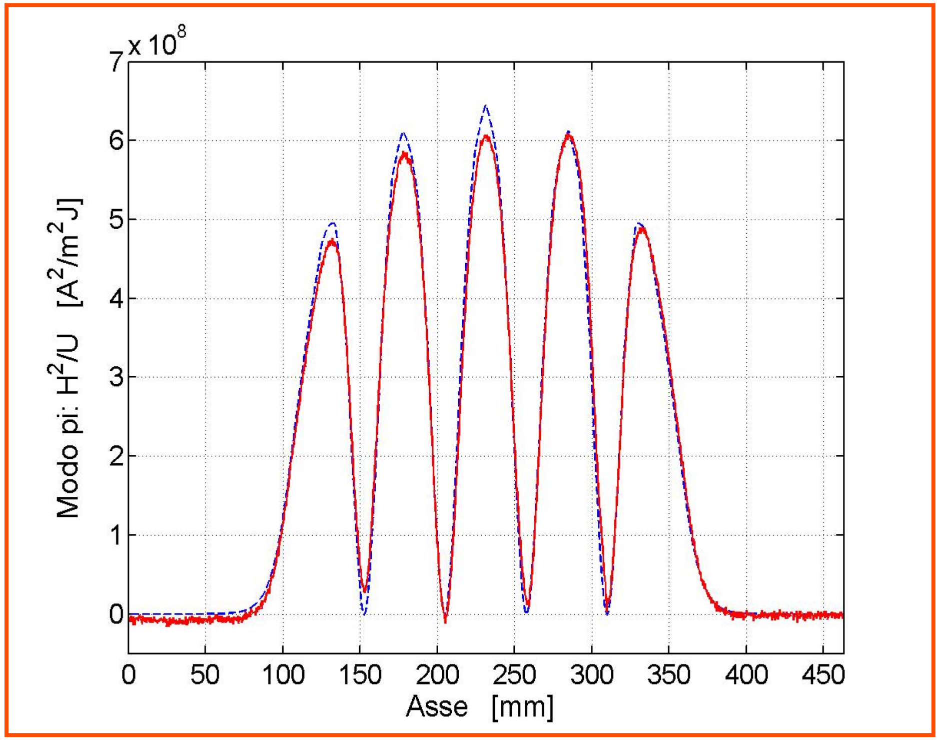
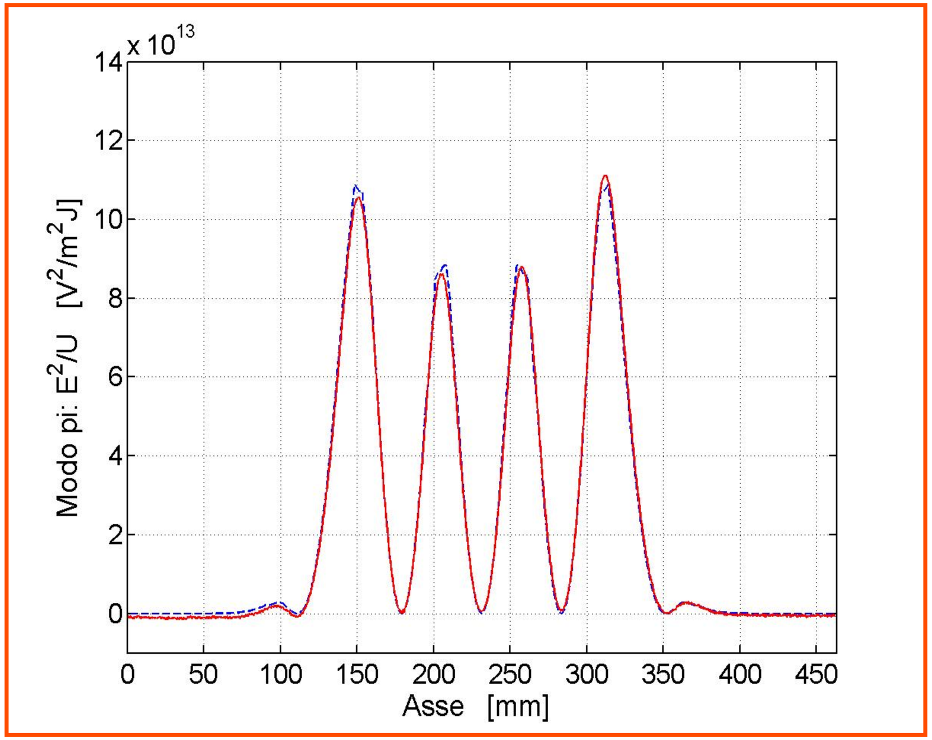
Andrea.Mostacci@uniroma1.it

Field on the beam axis





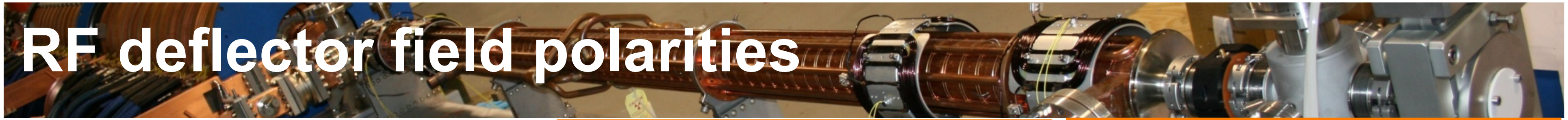


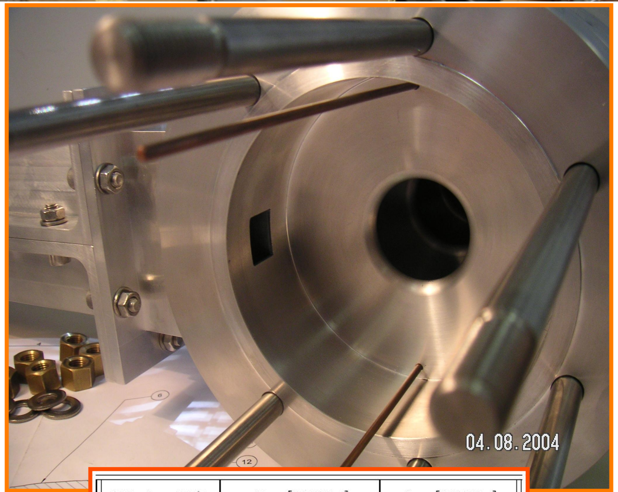
Simulations (HFSS) **Measurements (bead pull)**

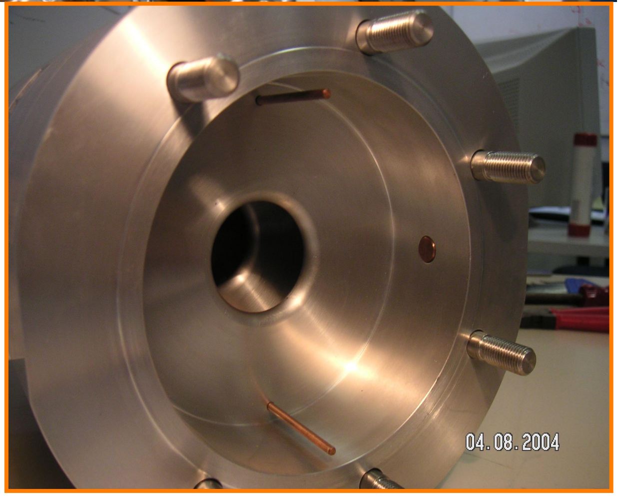
Courtesy of L. Ficcadenti

Andrea.Mostacci@uniroma1.it

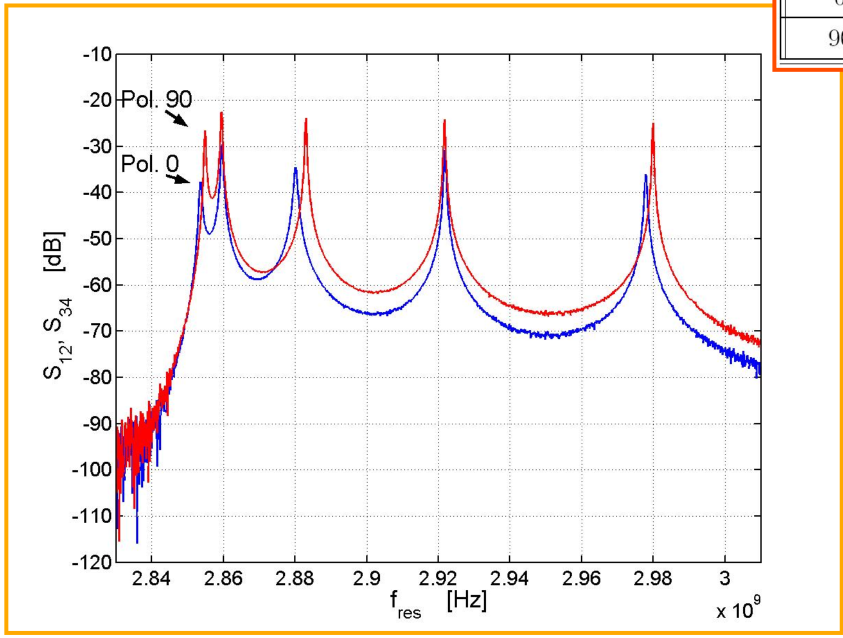
RF deflector field polarities

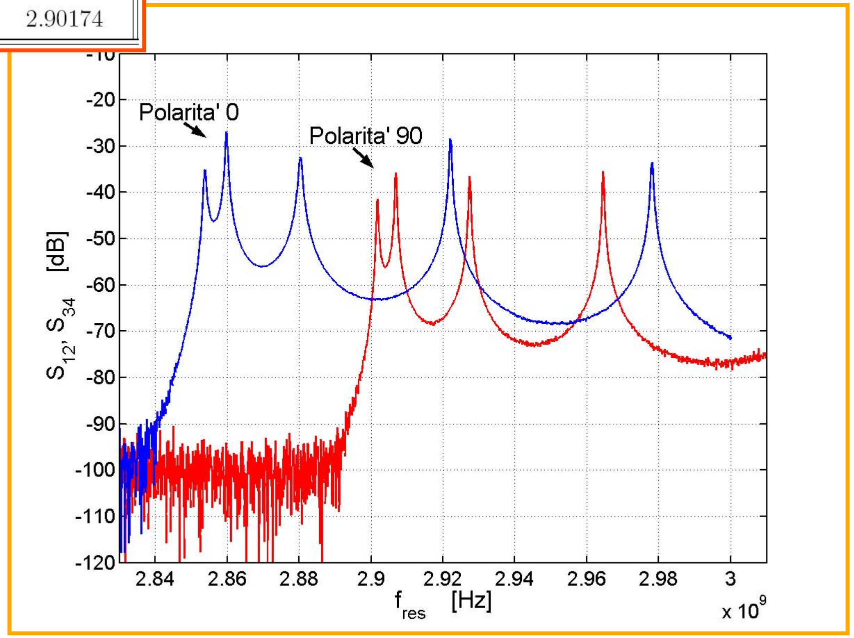






Polarità modo π	$f_{res}[GHz]$ senza barre	$f_{res}[GHz]$ con barre
0°	2.85358	2.85379
90°	2.85482	2.90174





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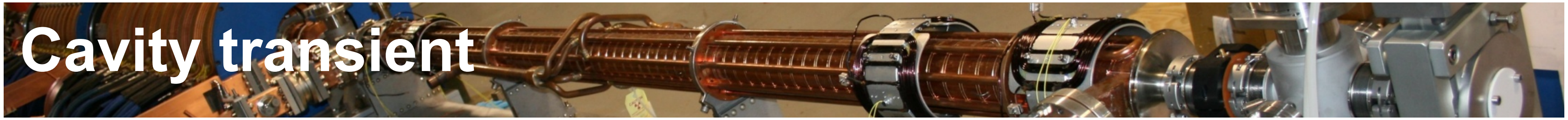
Andrea.Mostacci@uniroma1.it

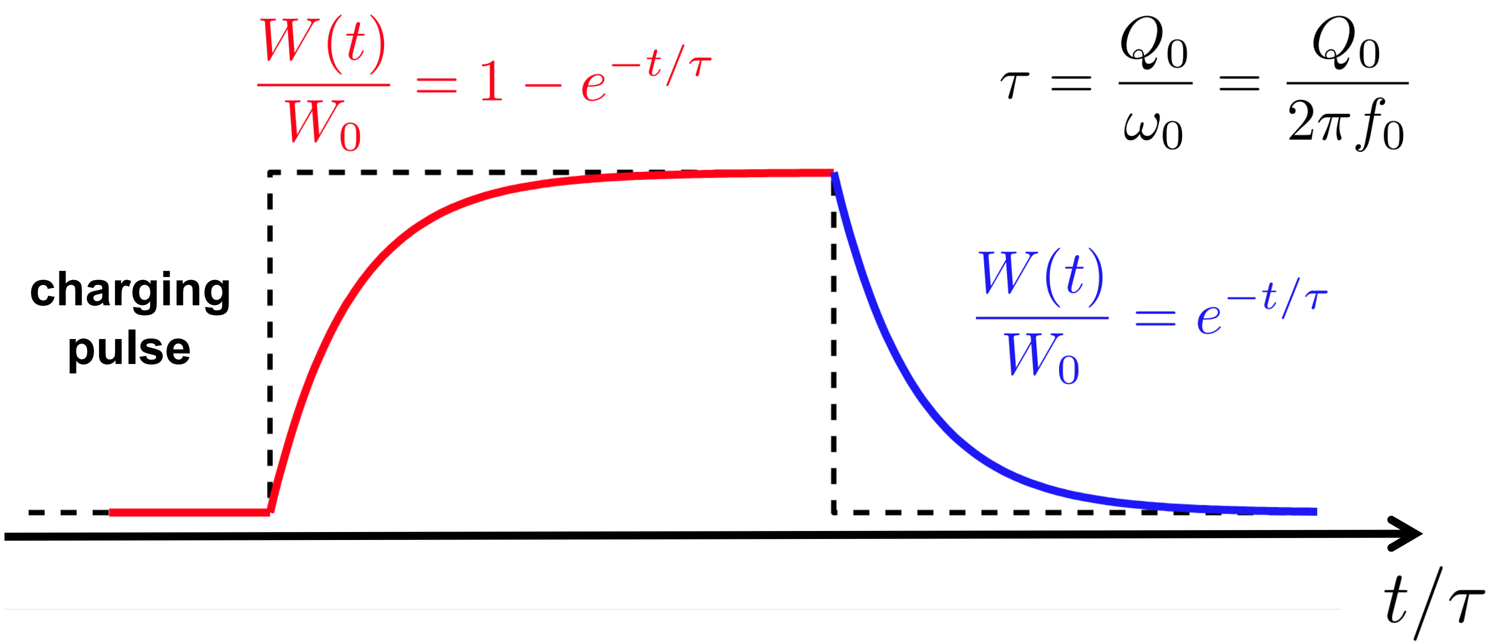
Cavity transient

ANDREA MOSTACCI



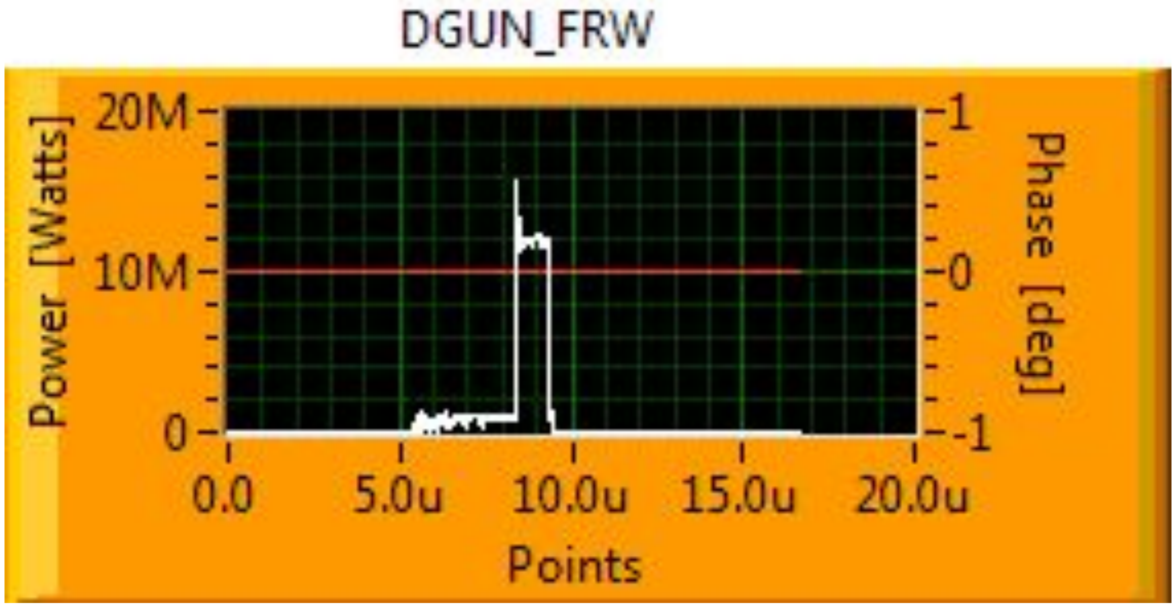
Cavity transient

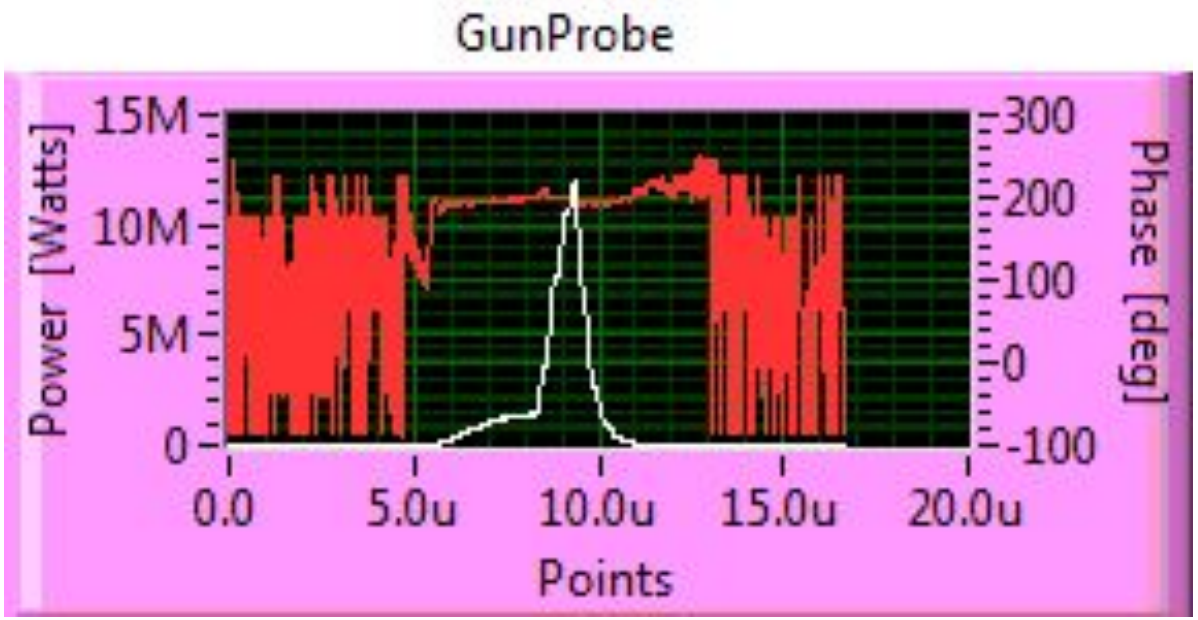
$$-dW = P_d dt = \frac{\omega_0 W}{Q_0} dt$$



Andrea.Mostacci@uniroma1.it

Cavity transient @ SPARC





Andrea.Mostacci@uniroma1.it